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Development of a trade-off function for robust optimization problems in design engineering

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Abstract

In industrial design processes, engineers and designers always need to perform compromises between many different design objectives. In particular, trades-off between nominal performance and design sensitivity have received increasing interests in the past few years. Robust design optimization methods focus on such conflicting issues in design engineering. Specific functions dedicated to the RDO framework, expressing the admissible compromises expected between nominal performance and design sensitivity have not yet stimulated much developments. The main purpose of this research work aims to develop a trade-off function to select among a set of alternatives, solutions which achieve rational compromises between design objectives. The design optimization model, composed by a model of the system behavior and a knowledge-based model, is formulated through observation, interpretation and aggregation functions. Such a procedure enables first to model preferences, provides a quality indicator for design solutions and finally turns the initial multiobjective optimization problem into a mono-objective problem which is solved stochastically by genetic algorithm. As an illustration of the trade-off approach, the method is used to achieve robust solutions for a side-impact crashworthiness problem.

1 Introduction

In the early stage of the design process, difficulties may arise from a limited knowledge of the system, from the high level of uncertainties and incomplete data due to fluctuating operating conditions, modeling precision, material properties or manufacturing tolerances for example [1]. Two conflicting issues are generally of interest in uncertain design problems: the optimization of the overall performance and then the minimization of the design variability also known as robustness. Robust design optimization (RDO) methods mainly focus on analyzing the trade-off between the improvement of the nominal performance and the reduction the design sensitivity. In quality engineering, G. Taguchy [2,3] attempts to minimize the effects of uncertainties on design sensitivity and performance without eliminating the source of variation by suggesting a “signal to noise” ratio to quantify the robustness of a design solution. However this method suffers from the impossibility to control compromises realized between levels of performance and sensitivity. Indeed according to his preferences, a designer needs to properly formulate such trades-off in order to achieve robust solutions.

Therefore, for a complete automation of trade-studies in robust design problem, this research work about RDO methodologies, integrates not only the robustness as defined by G. Taguchi, but also the robustness of the designer choice based on the expression of preferences and admissible compromises. It is a topical issue in design processes, since decisions must lead to the selection of a unique design alternative which will be manufactured and put on the market. Taking robust decisions is therefore fundamental by allowing designers to make the best choices at the earliest stages of the design process, even if it is constrained by deep uncertainties. Benefits come from decreasing the number of iterations between preliminary and detailed phases of the design process and increasing the level of reliability and speed of convergence towards the most preferred solution.

An efficient way to deal with RDO problems consists in tackling them as a multiobjective optimization problem [4]. Difficulties arise from comparisons between properties. Typically, an interpretation step is required to turn the numerical values of some performance criteria into levels of acceptability ranging from zero to one. Aggregation strategies are then used to build a single indicator reflecting the overall satisfaction level of acceptability of the design. Research works have focused in designing interpretation function such as utility or desirability functions, and aggregations function [5,6] to aggregate some objectives into a global score. However the development of a trade-off function dedicated to tackle RDO problems in engineering, haven’t yet stimulated much interests. Indeed this problem requires a particular class of function to model the perception of the designer when the robustness is expected. For example, a design solution is expected first to satisfy some criteria of performance and then to be invariant to uncertainty.

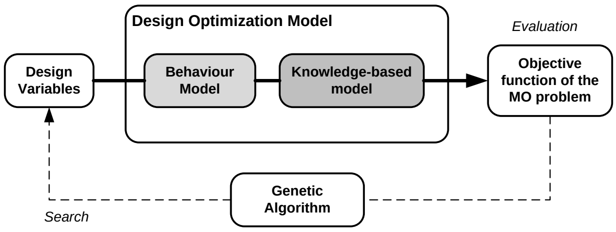


Fig. 1 Overview of the optimization model

The main purpose of this research work aims to develop a trade-off function to filter the Pareto front and achieve rational compromises between design objectives. A whole design optimization model composed by a model of the system behavior and a knowledge-based model involving interpretation and aggregation functions, is suggested to model preferences and provide a quality indicator for design solutions. The initial multiobjective optimization (MO) problem is thus turned into a mono-objective problem which is solved stochastically by developing a classical genetic algorithm. An overview of the developed methodology is presented in Fig. 1. As an illustration of the trade-off approach, the method is used to achieve robust solutions for a side-impact crashworthiness problem.

2 Observation, Interpretation and Aggregation (OIA)

In engineering design, technical solutions do not always meet the requirements and satisfy in unequal ways each design objective. The difficulty arises with the balancing act between the different levels of design objectives fulfillment. Therefore designers use their own judgment to express preferences on the design objectives and perform trade studies on the resulting design to converge toward the most preferred alternative.

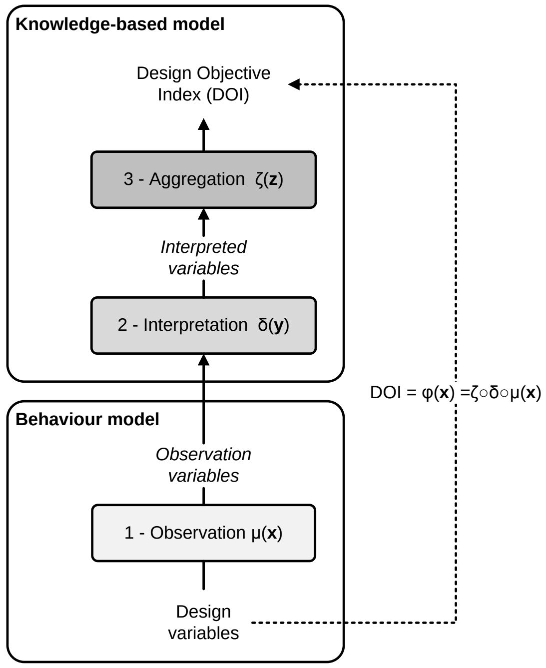


Fig. 2 Formulation of preferences with OIA

As illustrated in Fig. 2, such a process requires both objective and subjective knowledge. It is a three-step procedure which consists in first observing some relevant characteristic of the system, then interpreting their values into levels of acceptability according to constraints or requirement, and finally aggregating these interpreted values into a single global indicator. The designer's actions composed by observation, interpretation and aggregation (OIA) enable them to express preferences from a set of design variables. Each preference is intrinsically linked to a numerical value which conveys the level of fulfillment achieved by the chosen alternative. This value is the preference function. It brings together three kinds of functions which refer respectively to observation, interpretation and aggregation:

$$\varphi = \zeta \circ \delta \circ \mu \qquad (1)$$

where φ designates the preference function, μ is the observation function, δ is the interpretation function and ζ is the aggregation function. The OIA procedure has been already applied in the optimization of a two-stage flash evaporator [5,6].

2.1 Observation

Observation is the first thing that enables individuals to make judgment and so, to operate choices. During the design process, decisions about the values of design variables are often based on the observation of some relevant performance criteria of the system such as mass, efficiency, resources consumption, environmental impact or cost. The observation model is the behavior model of the system, combining relations and variables from physical, manufacturing, environmental and economical models. The observation model can be express as follows,

$$\mathbf{y} = \mu(\mathbf{x}), \quad \mathbf{x} \in \Omega \qquad (2)$$

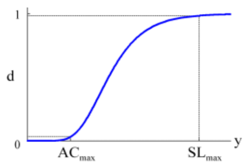
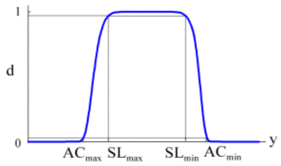
where μ is the observation model, y is the set of observation variables computed from a set of design variables **x** taken in the domain Ω.

2.2 Interpretation

The selection of an optimal design depends first on the satisfaction of constraints by the performance criteria, and then, on the overall level of performance reached by this solution. Thus interpretation aims to provide a same scale of value for criteria comparison according to their ability to satisfy constraints. Constraints are turned into objectives by mapping their value onto a function in the domain [0;1]. Observation variable values are thus associated to a level of acceptability. A level of acceptability close to zero usually leads to a non acceptable design alternative. A value closes to one means that the constraint has been completely satisfied. The alternative can be thus considered as acceptable. Moreover it becomes thus possible to provide the design problem formulation with flexibility by specifying how strict the constraints are. This whole knowledge-based model built on the designer judgment and experience is expressed through interpretation functions which are defined as a vectorial function of observation variables by:

$$\mathbf{z} = \delta(\mathbf{y}), \text{ with } \mathbf{z} \in [0,1] \qquad (3)$$

where δ represents interpretation functions and **z** is the set of interpreted variables in [0;1] derived from the set of observation variables **y**. In this paper we use a widely adopted class of interpretation functions based on the concept of desirability. In [7], Harrington proposed a multicriteria optimization scheme for industrial quality management based on what he called desirability functions. These functions are represented in Tab. 1.

One-sided (Max. form)	
	$d(y) = \exp(-\exp(\beta + \alpha \cdot y))$ $\begin{cases} d(SL_{max}) = 0.99 \\ d(AC_{max}) = 0.01 \end{cases}$
Two-sided (Target form)	
	$d(y) = \exp\left(-\left\lvert\frac{2 \cdot y - (U + L)}{U - L}\right\rvert^{n}\right)$ $\begin{cases} L = (AC_{max} + SL_{max})/2 \\ U = (SL_{min} + AC_{min})/2 \\ d(SL_{max}) = 0.99 \end{cases}$

Tab. 1 Harrington desirability functions

2.3 Aggregation

As a general rule every observation variable and hence, every interpretation variable, is related to one or more design objectives. While a constraint is a functional or technical requirement that the system must satisfy, a design objective can be defined as a general target or a task-specific constraint that the system should meet. Reducing manufacturing costs or environmental impacts are both usual relevant objectives in engineering design. Aggregation processes consist in first identifying performance criteria related to a same design objective, and then, combining their different levels of acceptability, or desirability, into one single indicator. This indicator, called Design Objective Index and denoted DOI in this paper, quantifies the overall desirability level reached by a design alternative in regard to one particular design objective. We express aggregation functions as:

$$DOI = \zeta(\mathbf{z}) \ , \ \text{with } DOI \in [0;1] \qquad (4)$$

The design objective index is derived from the desirability index introduced by Derringer in [8]. He suggested to aggregate desirability functions according to a weighted geometric mean to tackle multicriteria optimization problems. Later, Kim and Lin [9] modified this approach by computing the desirability index as the minimum of the desirability values. These two aggregation formulations are designated respectively as aggressive and conservative design strategy in the Method of Imprecision (MoI) developed by Anthonsson in [10,11]. Conservative design strategy is used while trade off aims to improve the design by increasing the desirability level of the worst aspect and decreasing the overall desirability level. In this case, the aggregation function is the minimum function:

$$\zeta(\mathbf{z}) = \min.\{z_1, \ldots, z_n\} \qquad (5)$$

where $\mathbf{z}$ is a set of n interpreted variables. Conversely, aggressive design strategy consists in increasing the overall level of desirability by slightly reduced the compensatory effects on the lowest property. Sometimes the resulting design is strongly hampered by a single criterion which is much more difficult to satisfy than all others. Therefore relaxing this constraint enables an improvement of the overall design. This strategy is expressed as a geometric mean aggregation:

$$\zeta(\mathbf{z}) = \prod_{i=1}^{n} z_i^{w_i} \qquad (6)$$

Each component of the aggregation can be provided with a numerical weight w_i which reflects a partial order of importance between preferences. Weight assignments in aggregation processes are not tackled in this paper and therefore, all weights are taken as equal, i.e. we suppose that all components of the aggregation are equally preferred. Conservative and aggressive aggregation strategies can be applied successively to combine many design objectives into a single global main objective.

3 Development of a trade-off function

In the continuity of the OIA procedure, we have developed a new objective function, specific to robust design problems, i.e. while designers performed trade studies between first improving the overall level of nominal performance and then reducing the design sensitivity. The awareness of such compromises is a main source of concern in design engineering since they enable to make robust decisions and then, to converge quickly towards the most preferred alternatives.

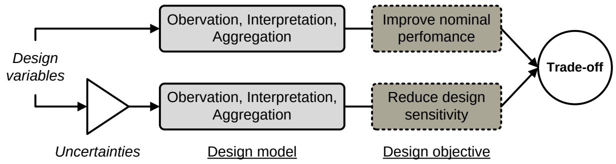


Fig. 3 Principle of the trade-off approach

The developed approach is exposed in Fig. 3. We use the OIA procedure to formulate two design objectives related to robustness. While the first one aims to improve the nominal performances of the system, the other one concerns the minimization of the sensitivity of the design to perturbations. Then, instead of performing another aggregation as suggested in part 2.3, a trade-off function is thus introduced to filter a set of solutions according to their ability to improve or degrade compromises linked to robust design.

3.1 Trade-off function and trade-off indicator

The trade-off function has been designed to provide a suitable measurement for the relative sensitivity of a choice from a set of alternatives, by quantifying the improvement or degradation in the compromise between two preferences when one of both is prevailing. In a robust deign framework, it is often rational to first achieve a satisfying level of performance and then to reduce the sensitivity of the solution. Achieve a good level of nominal performance and reduce variability can be considered as two design objectives in this case. The trade-off function is a three parts function defined as:

$$\tau:\begin{cases}[0,1]^2 \mapsto [-1,1] \\ (u,v) \to A\cdot\left(1-u^n-\left(1-k^n\right)\cdot v^n\right)+B\end{cases}$$

with $t = 1-u^n-\left(1-k^n\right)\cdot v^n$

$$\begin{cases}(1)\ \text{if } u\geq k \text{ and } t\leq 0 \text{ then } A=-1/\left(1-k^n\right),\ B=0 \\ (2)\ \text{if } u\geq k \text{ and } t>0 \text{ then } A=-1,\ B=0 \\ (3)\ \text{if } u<k \text{ and } t>0 \text{ then } A=-u/k,\ B=\left(u-k\right)/k\end{cases}$$

where (u,v) are two normalized preferences such as u is dominant compared to v. The preference u refers to the objective of improving the nominal performance whereas the preference v designates the objective of reducing the variability of the design. Parameters k and n are specification parameters of the trade-off function. These parameters are related to an iso trade-off of reference given by τ (u, v) = 0.

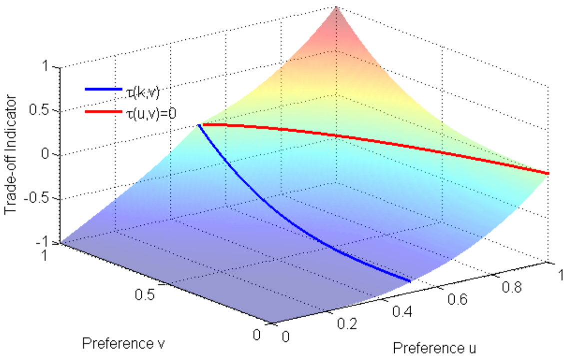


Fig. 4 Trade-off function specified with k = 0.5 and n = 3

The iso trade-off function which is further detailed in part 3.2, represents all the alternatives considered as equally preferred by the decision-maker. In Fig. 4, a trade-off function specified with k=0.5 and n=3 has been represented. The trade-off function ranks a set of solutions alternatives according to their ability to constitute a better, equivalent or worse compromise than the trade-off initially defined by the iso trade-off function. Derived from the desirability index, we introduce here the trade-off indicator TI defined as:

$$TI = \tau\left(u,v\right) \qquad (7)$$

From a couple of preferences, the trade-off function thus provides a relative measure expressing the improvement, or the degradation in the compromise between two candidate solutions. Therefore such an indicator traduces a partial order relation and is associated to a particular set of solutions. Considering a set of design alternatives evaluated on two preferences, a positive trade-off indicator indicates an improvement in the compromise, and so a more relevant choice, whereas a negative trade-off indicator traduces a degradation and so a worst choice. The iso trade-off function is associated to a trade-off indicator equal to zero since it represents equivalent choices and therefore a conservation of the trade-off.

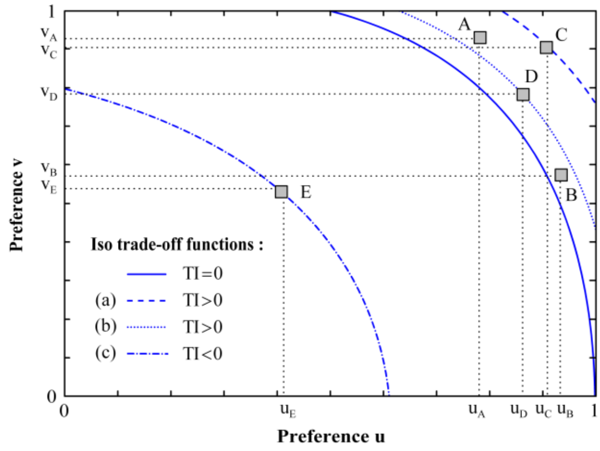


Fig. 5 Iso trade-off curves

In Fig. 5 alternatives A and B express a compensatory situation where no rational decision can be taken. The first example (a) deals with trade-off improvement. Alternative C is equivalent to alternative A in regard to preference v but, it is also better in regard to preference u. Compared to alternative B, alternative C is equivalent in regard to preference u but, better in regard to preference v. Therefore alternative C constitutes a better choice than both A and B. The second example (b) deals with the conservation of the trade-off. According to the value obtained for each preference, it is actually not possible for the decision-makers to operate a rational choice between alternatives A, B and D, which can be considered as equivalent. The last example (c) refers to trade-off degradation. As the alternative E is less preferred than both alternatives A and B for the two preferences u and v, then it constitutes the worst compromise. The partial orders of relations associated to these examples are reported in Tab. 2.

Trade-off cases	Partial order relations
a –Improvement	$\begin{cases}u_A \square u_C, & v_A \approx v_C \\ u_B \approx u_C, & v_B \square v_C\end{cases} \Rightarrow C \succ (A,B)$
b – Conservation	$\begin{cases}u_A < u_D, & v_A > v_D \\ u_B > u_D, & v_B < v_D\end{cases} \Rightarrow D \approx (A,B)$
b – Degradation	$\begin{cases}u_A > u_E, & v_A \square v_E \\ u_B \square u_E, & v_B \approx v_E\end{cases} \Rightarrow E \prec (A,B)$

Tab. 2 Trade-off and partial order relations

These particular properties of the trade-off function enable to provide a consistent ranking of the possible alternatives, even in borderline cases while all alternatives achieve the same level of nominal performance, or the same level of design sensitivity or equivalent levels.

3.2 Iso trade-off function

The general expression of the trade-off function given in part 3.1, is mainly derived from the definition of the iso trade-off function. This function is derived from preference modeling in decision theory. Preferences and their logical properties refer to a choice between some alternatives and the possibility to rank these alternatives according to degrees of satisfaction, utilities, desirabilities or other evaluation criteria. The iso trade-off function modeled an indifference relation which can also been regarded as an equivalence relation. Such a function represents all the alternatives considered as equally preferred by the decision-maker. Let's consider two preferences (u,v) as introduced previously. Typically, the ideal decision made from alternatives concerns the one which achieves the best preferences for both u and v. Such cases are rare in real decision problems. Designers often face to compensatory situations and have to operate selections among a set of design solutions. According to the trade-off approach, if preference u is required to keep a minimal value, then the compromises between u and v can be expressed as the maximal degradation of u allowed by the designer to improve the preference v. The iso trade-off function is based on this statement and considers every solution that verifies the following equation as equally preferred:

$$1-u^n-v^n\left(1-k^n\right)=0 \quad \text{with } \begin{cases} u>k, & k\in[0,1] \\ n\in\Re_+^* \end{cases} \tag{8}$$

where k and n are the two specification parameters required to adjust the shape of the function of the decision problem. Parameter k gives the minimal admissible value reached by preference u in order to increase the value of preference v from zero to one. Parameter n is used to refine the expression of the compromise expected between the two preferences. Increasing the value of n makes it possible to be more and more restrictive on the minimal admissible value taken by preference u. Through this formulation we suggest that a design solution satisfying the objective of performance, i.e. such as u=1, but not robust (v=0) is equivalent to a solution less performing (u=k) but much more robust (v=1).

4 Application: trades-off for vehicle side-impact crashworthiness

As an illustration of this trade-off approach, the method is used to achieve robust design solutions for the car side-impact crashworthiness problem already discussed in [12,13,14] and presented on Fig. 6. This multiobjective optimization problem aims to design a vehicle structure under uncertainties, which not only satisfies a number of safety conditions imposed by the crashworthiness regulations, but also minimizes the overall weight of the car.

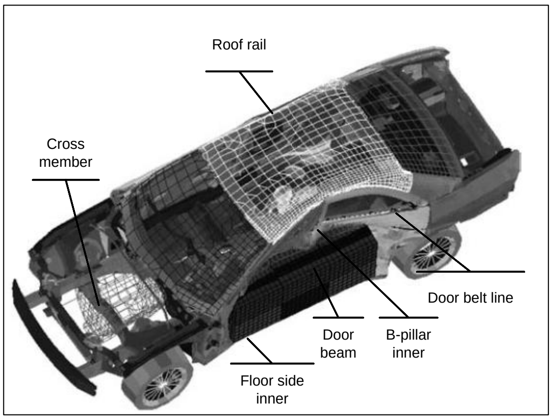


Fig. 6 Vehiccle side impact model adpated from [13]

4.1 Description of the vehicle side-impact crashworthiness problem

Initially the objective of the side-impact crashworthiness problem is to reduce the vehicle weight while satisfying safety criteria, by adjusting the values of nine design variables presented in Tab. 3 and illustrated on Fig. 6. Design variables from x_1 and x_7 refer to the thickness of the different parts of the vehicle structure namely the B-pillar inner, the B-pillar reinforce, the floor side inner, the cross member, the door beam, the door belt line and the roof rail. These variables are supposed to be continuous. Design variables x_8 and x_9 refer respectively to the material properties (medium-strength steel and high-strength steel) of the B-pillar inner and floor side inner. They are considered as discrete variables. In addition to this nine design variables there are two other variables which represent the barrier hitting height x_{10} and the barrier hitting position x_{11}. In this study we take these two last variables equal to zero. Furthermore variables from x_1 to x_7 are supposed to have a normal distribution with a standard deviation equal to 0.03.

D. variables	Type	Domain Ω	Dist.	Std.
x_1	continuous	[0.5 ; 1.5]	$\mathcal{N}$	0.03
x_2	continuous	[0.5 ; 1.5]	$\mathcal{N}$	0.03
x_3	continuous	[0.5 ; 1.5]	$\mathcal{N}$	0.03
x_4	continuous	[0.5 ; 1.5]	$\mathcal{N}$	0.03
x_5	continuous	[0.5 ; 1.5]	$\mathcal{N}$	0.03
x_6	continuous	[0.5 ; 1.5]	$\mathcal{N}$	0.03
x_7	continuous	[0.5 ; 1.5]	$\mathcal{N}$	0.03
x_8	discrete	{0.192 ; 0.345}	-	-
x_9	discrete	{0.192 ; 0.345}	-	-

Tab. 3 Design variables of the vehicle side-impact crashworthiness problem

The response variables of the side-impact crashworthiness model are the observation variables of the OIA formulation. They concern the overall weight of the car (y_1) and the safety performances involving the occupant and structural damages. These safety criteria concern the abdomen load (y_2); the chest injury caused by the deformation of soft tissues measured at three different places on the torso (viscous criterion; y_3 - y_4 - y_5); the rib deflections measured at the upper, middle and lower chest area (y_6 - y_7 - y_8) and the possible tear in the cartilage connecting the right and left pubic bone (public symphisis force, y_9). The behavior of the vehicle structure includes the velocity of the B-pillar at the middle point (y_{10}) and the velocity of the front door at the B-pillar (y_{11}). The FE simulation models and the statistical analysis performed to derive the response surface model are described in [13]. The response surfaces giving the weight of the vehicle and the safety criteria are expressed as follow:

$$y_1 = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + \ldots 2.73x_7$$

$$y_2 = 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 + \ldots 0.01343x_6x_{10}$$

$$y_3 = 0.261 - 0.0159x_1x_2 - 0.188x_1x_8 - 0.019x_2x_7 + \ldots 0.0144x_3x_5 + 0.0008757x_5x_{10} + 0.08045x_6x_9 + \ldots 0.00139x_8x_{11} + 0.00001575x_{10}x_{11}$$

$$y_4 = 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 + \ldots 0.03099x_2x_6 - 0.018x_2x_7 + 0.0208x_3x_8 + 0.121x_3x_9 - \ldots 0.00364x_5x_6 + 0.0007715x_5x_{10} - 0.0005354x_6x_{10} + \ldots 0.00121x_8x_{11}$$

$$y_5 = 0.74 - 0.61x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} - \ldots 0.166x_7x_9 + 0.227x_2x_2$$

$$y_6 = 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_5x_{10} + 6.63x_6x_9 - \ldots 7.7x_7x_8 + 0.32x_9x_{10}$$

$$y_7 = 33.86 + 2.95x_3 + 0.1792x_{10} - 5.057x_1x_2 - 11.0x_2x_8 - \ldots 0.0215x_5x_{10} - 9.98x_7x_8 + 22.0x_8x_9$$

$$y_8 = 46.36 - 9.90x_2 - 12.9x_1x_8 + 0.1107x_3x_{10}$$

$$y_9 = 4.72 - 0.5x4 - 0.19x_2x_3 - 0.0122x_4x_{10} + \ldots 0.009325x_6x_{10} + 0.000191x_{11}x_{11}$$

$$y_{10} = 10.58 - 0.674x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10} - \ldots 0.0198x_4x_{10} + 0.028x_6x_{10}$$

$$y_{11} = 16.45 - 0.489x_3x_7 - 0.843x_5x_6 + 0.0432x_9x_{10} - \ldots 0.0556x_9x_{11} - 0.000786x_{11}x_{11}$$

These expressions enable to provide the mathematical formulation for the initial optimization model P_1 of the vehicle side-impact crashworthiness:

$$\begin{aligned}
P_1:&\quad \min.\ y_1\\
\text{Sub. to}:&\quad y_2 \le 1.0\ \text{kN}\\
&\quad y_3, y_4, y_5 \le 0.32\ \text{mm}\\
&\quad y_6, y_7, y_8 \le 32\ \text{mm}\\
&\quad y_9 \le 4.0\ \text{kN}\\
&\quad y_{10} \le 9.9\ \text{mm}\cdot\text{ms}^{-1}\\
&\quad y_{11} \le 15.7\ \text{mm}\cdot\text{ms}^{-1}\\
&\quad \mathbf{x} \in \Omega \text{ with } x_{10} = x_{11} = 0
\end{aligned}$$

The trade-off optimization problem is mainly derived from this formulation. The objective is to reach the best compromise between the nominal performance involving the minimization of the weight and the respect of safety criteria, and the design stability. In this study, the design stability is based on the probability that a constraint is violated according to safety criteria.

4.2 Formulation of the optimization problem through the OIA procedure

According to the methodology developed on part 2, the initial optimization model P_1 of the vehicle side-impact crashworthiness is formulated through OIA processes and completed by including design stability and trades-off. Fig. 7 illustrates the whole design model derived from the trade-off approach and OIA procedure. Starting from a global design objective, the designer can structure the design problem by setting actions which refer to observation, interpretation and aggregation. The main design problem is thus divided into objectives and sub-objectives which have to be satisfied. Ones can notice that the computing process starts by first evaluating the observation variables, then the value reached by the different objectives and finally the global design objective. The main design objective of the modified design problem is to achieve design solutions which satisfy trades-off between nominal performance and design stability. This action is modeled using a trade-off function between two preferences related to two design objectives: improve the overall level of performance (DOI_1) and reduce the variability of the design under uncertain data (DO_2). The design objective linked to performance is defined by the fulfillment of two sub-design objectives related respectively to the minimization of the vehicle weight and the satisfaction of safety conditions. This is modeled using a weighted geometric mean aggregation. The optimization of the overall weight is directly given by the interpretation of the observation variable y1 through an ones-sided Harrington desirability function specified with an absolute constraint equal to 30 and a soft limit taken at 20. The safety objective is obtained by satisfying every safety criteria and thus modeled by aggregating binary values (0 or 1) provided by strict threshold interpretation functions. Therefore, through an OIA process, the initial optimization model P_1 of the vehicle side-impact crashworthiness is now formulated as P_2 by:

$$\begin{aligned}
P_2:&\quad \max.\ DOI_1\\
\text{Sub. to}:&\quad \mathbf{x} \in \Omega \text{ with } x_{10} = x_{11} = 0
\end{aligned}$$

Such a formulation has turned the multiobjective optimization problem into a mono objective problem. Constraints and preferences are formulated inside the objective function through interpretation and aggregation steps, and consequently, they are not explicit in the formulation of the optimization problem, but intrinsic to its definition. However both formulations P_1 and P_2 remain equivalent and share the same optimal solution presented in the following.

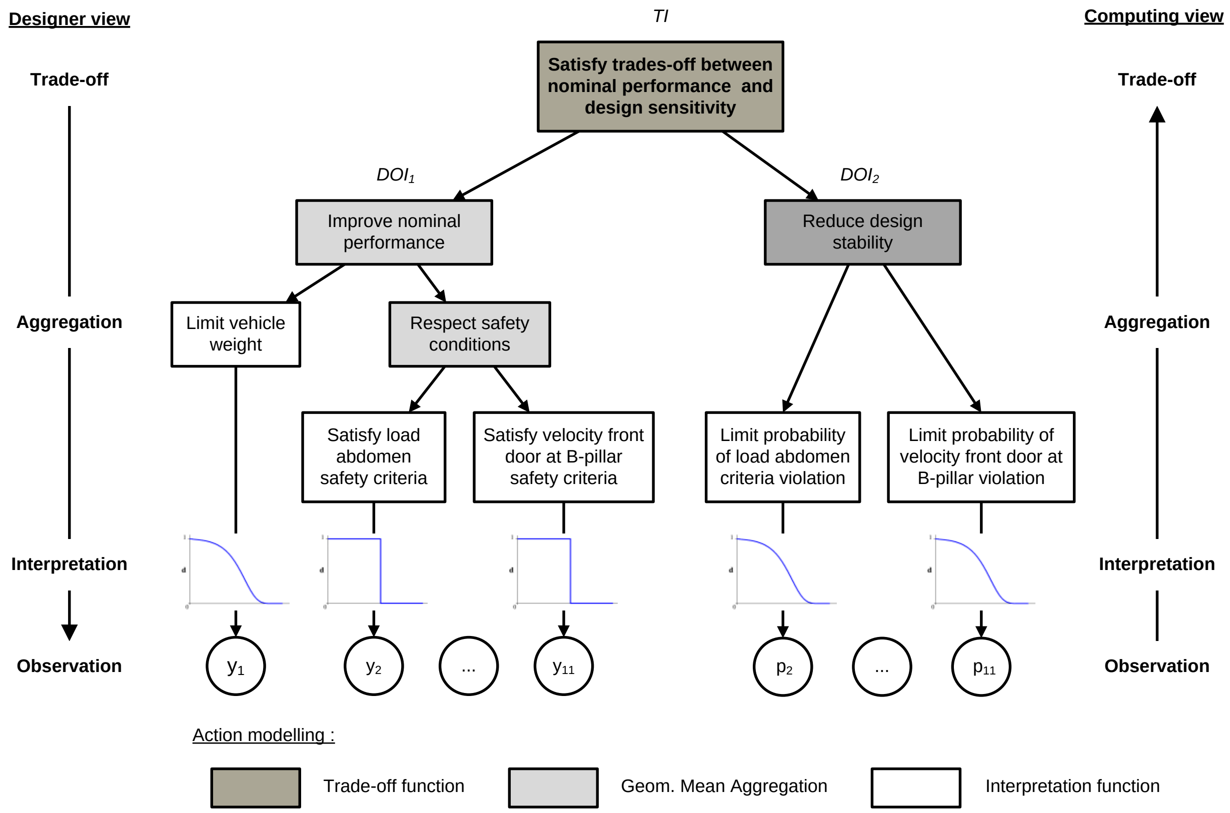


Fig. 7 Design optimization model of the vehicle side-impact crashworthiness

The design objective linked to the minimization of the design stability deals with the probability that constraints are satisfied or not by safety criteria, while design variables (x_1-x_7) are randomly disturbed. This action is modeled using a weighted geometric mean aggregation composed by the interpretation of the constraint violation probability p for the observation variable y. The interpretation function used is the one-sided Harrington desirability function. The hard constraint is set according the admissible probability of non-respect of the constraint. Finally the modified design optimization model P3 of the vehicle side-impact crashworthiness is formulated as follow:

$$P_3: \quad \max.\, TI$$
$$\text{Sub. to}: \quad \mathbf{x} \in \Omega \text{ with } x_{10} = x_{11} = 0$$

In this research work, we consider that all components involved in the different aggregation processes are equally preferred, and thus the corresponding weights are equal. The design solutions related to P_2 and P_3 are discussed in part 4.4.

4.3 Numerical solving

The numerical solving problem has been addressed by developing a classical genetic algorithm [15]. This nature-inspired algorithm takes into account a set of finite alternative solutions which are evaluated and ranked according to their fitness, i.e. the objective function value. The weakest candidates are then eliminated before a new set is created. Four operators, with different occurrence probabilities, ensure the convergence of the set toward the optimal point: the tournament selection, the cross-over, mutation and climber operators. The optimization process has been performed with a population of 240 individuals with an elitism strategy, a tournament size of 2 with an initial selection pressure of 50%, a crossing probability of 80%, a mutation probability of 10% and a climbing probability of 20%.

For the optimization problem P_3, the fitness function has been modified to handle uncertainties. First each individual is evaluated 1000 times using Monte Carlo simulations. Moreover as the trade-off function is relative to a particular population, the design objective indexes DOI_1 and DOI_2 must be normalized by their maximum value over the current population. But in this study, for the design objective linked to the performance, we take as reference the optimal design solution of the optimization problem P2. In other words, we suppose that the nominal performance of this design which is obviously the highest is expected to be decreased in order to improve its robustness.

4.4 Results and discussion

The optimization problem P_2 is first processed. The optimal solution is obtained in 344 generations and is reported in Tab. 4. This solution presents an overall level of desirability of 0.9314 for the design objective linked to the nominal performance (DOI_1). This design is fortunately the same as the one obtained by deterministic method in [14] for the optimization problem P_1. However this optimal design is not a robust optimal solution in particular for the safety criteria. In fact the observation variables y8, y9 and y11 are equal to their admissible values. Therefore a slightest variability on the design variables makes the safety design objective to be no longer satisfied. Finally the optimization problem P_3 is processed with absolute probabilistic constraints of 90% and 99%. In this study we have chosen to set k=0.5 and n=3. The optimal designs solutions provides by P_2 and P_3 are denoted respectively as S_2 and S_3.

Designs solutions :	S_2	$S_{3,90\%}$	$S_{3,99\%}$
TI $_{90\%}$	0.000	<u>0.899</u>	0.793
TI $_{99\%}$	0.000	-0.017	<u>0.793</u>
Design variables:			
x_1	0.5000	0.5008	0.5000
x_2	1.2258	1.3051	1.3605
x_3	0.5000	0.5000	0.5003
x_4	1.2071	1.2742	1.3367
x_5	0.5000	0.5939	0.6574
x_6	1.4932	1.5000	1.5000
x_7	0.5000	0.5018	0.5000
x_8	0.3450	0.3450	0.3450
x_9	0.1920	0.1920	0.1920

Tab. 4 Optimal designs for the side-impact crashworthiness problem with different probabilistic constraint

According to this formulation of the design optimization problem, the developed trade-off approach enables to achieve solutions which present a more robust compromise than the initial solution S_2. Tab. 4 represents the optimal designs for the side-impact crashworthiness problem tackled with different probabilistic constraint. The solution of reference S_2 gets a trade-off indicator equal to zero since looking at Tab. 5 and Tab. 6, this solution has the highest score for the performance objective (DOI_1) but doesn't meet the conditions imposed by the stability objective (DOI_2).

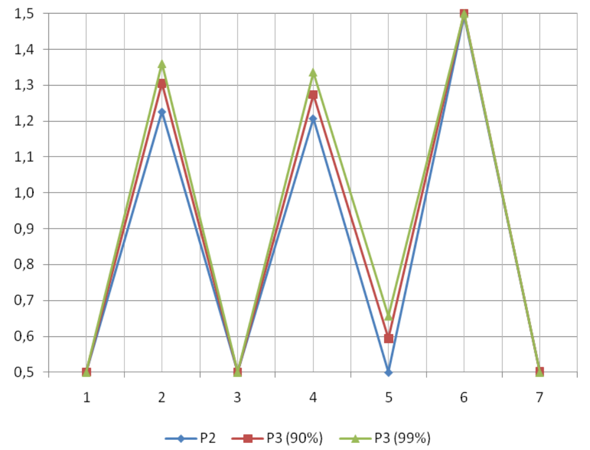


Fig. 8 Seven first design variables for the side-impact crashworthiness problem using different approaches

Fig. 8 illustrates the design variables of the side-impact crashworthiness problem brought back from the initial domain Ω into the range [0;1]. Looking at the design variables on Tab. 4 and Fig. 8, it appears that the three design configurations are very close. Obviously the variables x_2, x_4 and x_5 are the most sensible and thus enable to jump from a design solution to another. Consequently, starting from the reference solution S_2, it is thus possible to perform slight modifications of the design variables values to improve the compromise between nominal performance and design stability of such a solution.

Designs solutions :	S_2	$S_{3,90\%}$	$S_{3,99\%}$
DOI_1	<u>0.931</u>	0.879	0.818
Observ. variables:			
Min. y_1	23.192	24.166	24.892
$y_2 \leq 1.00$	0.564	0.495	0.438
$y_3 \leq 0.32$	0.234	0.233	0.233
$y_4 \leq 0.32$	0.247	0.250	0.253
$y_5 \leq 0.32$	0.289	0.286	0.286
$y_6 \leq 32.0$	28.887	28.720	28.614
$y_7 \leq 32.0$	27.319	26.806	26.468
$y_8 \leq 32.0$	31.999	31.210	30.665
$y_9 \leq 4.00$	4.000	3.959	3.922
$y_{10} \leq 9.90$	9.342	9.261	9.206
$y_{11} \leq 15.7$	15.698	15.576	15.496

Tab. 5 Fulfillment of the nominal performance objective for the side-impact crashworthiness problem

For the optimization problem specified with a 90% probabilistic constraint, the best trade-off indicator equals 0.899 and thus, the design solution $S_{3,90\%}$ traduces a great improvement of the compromise between nominal performance and design variability. From Tab. 5 and Tab. 6, we can observe that a compromise of 0.05 on the nominal performance objective enables to improve the design stability objective from 0 to 0.961. In term of performance, this is traduced by a 4% weight increasing while satisfying the safety criteria. However, in the same time, the risks of constraint violations have been reduced from 50% to 4 % for the safety criteria y_8, y_9 and y_{11}. While increasing the probabilistic constraint from 90% to 99%, the solution $S_{3,90\%}$ is no longer optimal since its score for the stability objective is null. The associated trade-off indicator is thus negative and so such a solution represents a worst choice than S_2 according to the problem specifications. Consequently, a new optimal solution $S_{3,99\%}$ is reached. As it is more difficult to satisfy the new requirements set on the stability objective, a higher decreasing on the nominal performance is required to improve the trade-off indicator. This solution is safer than the other by strongly minimizing the risks of safety constraints violations.

In this side-impact crashworthiness problem, a modification of the shape of the trade-off function by increasing or decreasing the values of parameters k and n doesn't modify the optimum. In fact, as there is a design solution with a very high trade-off indicator among all possible alternatives, then the optimization process converges toward this solution whatever the trade-off function specification, unless to impose a very restrictive value to k such as k = 0.95 and in this case, the optimization algorithm converges toward S_2.

Designs solutions :	S_2	$S_{3,90\%}$	$S_{3,99\%}$
$DOI_{2,\,90\%}$	0.000	0.969	0.990
$DOI_{2,\,99\%}$	0.000	0.000	0.990
Probability:			
$p(y_2 \leq 1.00)$	1,00	1,00	1,00
$p(y_3 \leq 0.32)$	1,00	1,00	1,00
$p(y_4 \leq 0.32)$	1,00	1,00	1,00
$p(y_5 \leq 0.32)$	1,00	1,00	1,00
$p(y_6 \leq 32.0)$	1,00	1,00	1,00
$p(y_7 \leq 32.0)$	1,00	1,00	1,00
$p(y_8 \leq 32.0)$	0,49	0,96	1,00
$p(y_9 \leq 4.00)$	0,48	0,96	1,00
$p(y_{10} \leq 9.90)$	1,00	1,00	1,00
$p(y_{11} \leq 15.7)$	0,51	0,98	1,00

Tab. 6 Satisfaction of the design stability objective for a 90% and 99% probabilistic absolute constraint

5 Conclusion

In this paper, a new objective function, based on the trade-off between nominal performance and design sensitivity has been developed to tackle robust design optimization in engineering problems. In this framework, an original OIA procedure based on observation, interpretation and aggregation functions is suggested to provide the designer with a convenient formulation for expressing his preferences and qualifying particular design alternatives.

The salient point of this research work is to develop a trade-off function to filter a set of design solutions and to achieve rational compromises between design objectives. It results in an objective function involving not only the optimality and sensitivity of the solution, but also the compromises expected by the designer. The trade-off function qualifies the degradation of the nominal performance allowed by the designer to reduce the design sensitivity of the preferred solution. The formulation of the trade-off function used in this paper is based on the specification of level curves called iso-trade-off functions. The trade-offs achieved by design alternatives are thus evaluated according to their capability to satisfy, degrade or improve the initial compromise.In the last section, a benchmark design problem based on vehicle side-impact crashworthiness had been presented. Through this case of application, the developed trade-off approach slightly modifies an initial solution of reference to achieve more robust design solutions. In this case the robustness of the design is evaluated according the degree of confidence in the strict respect of safety criteria.

The trade-off approach coupled with an OIA scheme seems promising to tackle RDO problems. However the interaction with the designer through the specification of the parameters of the trade-off function must be further investigated in future research work.

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