*Proceedings of the IMProVe 2011*
*International conference on Innovative Methods in Product Design*
*June 15th – 17th, 2011, Venice, Italy*

# A novel method for sensitivity analysis and characterization in integrated engineering design

*A.O. Andrisano* [(a)], *M. Ansaloni* [(a)], *F. Leali* [(a)], *M. Pellicciari* [(a)], *A. Vergnano* [(a)]

[(a)] Department of Mechanical and Civil Engineering – University of Modena and Reggio Emilia (Italy)

## Article Information

Keywords:
*Robust Design,*
*Engineering Process,*
*Performance Sensitivity,*
*Sensitivity Maps*

Corresponding author:
*Matteo Ansaloni*
*Tel.: +390592056278*
*Fax.: +390592056126*
*e-mail: matteo.ansaloni@unimore.it*
Address: via Vignolese, 905/B – 41125, Modena (Italy)

## Abstract

Purpose:
*The present research work aims to analyze and characterize processes in terms of sensitivity of their performances. Robust design techniques, generally adopted for product and process optimization, are not suited for investigating sensitivity. Then a novel approach to such engineering problem needs to be proposed.*

Method:
*The developed method integrates and extends to the analysis of manufacturing and technological processes the Performance Sensitivity Distribution (PSD) theory, primarily introduced to provide analytical and geometric description of performance sensitivity for robotic mechanisms.*

Result:
*Such novel method, named Specialized PSD, starts from the clarification of the sensitivity analysis problem by defining key parameters, i.e. Design Variables (DVs), Design Parameters (DPs) and Performance Functions (PFs). According to the PSD theory, PF sensitivity is expressed in terms of deviations of DVs and DPs and it is geometrically described by a hyperellipsoid in the n-dimensional space. Sensitivity indexes are then introduced to assess PF variation for different combinations of DVs and DPs deviations. Regression Analysis is adopted to provide the mathematical description of PFs so the PSD theory is finally specialized to be applied in a process sensitivity analysis. Injection molding of a plastic specimen is finally investigated to validate the proposed method.*

Discussion & Conclusion:
*This work specialize the PSD theory for manufacturing and technological processes, extending its original field of application thanks to a novel approach to the analytical expression of the PFs. Moreover, when 2 or 3 parameters are considered, sensitivity indexes are graphically represented through tolerance maps of colour, so the method can be easily adopt for integrated design, especially in the early stage of product and process development.*

## 1 Introduction

Mechanical systems and engineering processes are commonly described by design parameters which determine, with their values and their variations, final performances and overall quality [1, 2]. Due to the complexity of a general engineering process or system is not possible, or economically feasible, to control and verify all the design parameters. Key Characteristics (KCs) have to be identified and investigated [3-4]. The KCs management and analysis need integrated engineering design methods based on advanced mathematical techniques and computer aided tools [5, 6].

Robust Design methods are widely adopted to find analytical relationships between performances and KCs and to define their values in order to achieve optimized performances. Well known examples are Taguchi Method, Surface Method, Artificial Neural Network, Fuzzy Regression and Desirability Function method [7, 8].

Such methods are very powerful for integrated engineering design but don't directly investigate the performance sensitivity, i.e. the measure of the performance variation due to the deviation of the KCs.

The robustness of a system increases when its sensitivity decreases so sensitivity analysis is particularly important in case of performance values closed to their limits, because they may be easily worsted by small parameters deviations in working conditions and design requirements may be not satisfied [9-11]. Due to its importance in delivering the final quality, sensitivity analysis might be included in the integrated engineering design workflow.

The present paper proposes the integration and extension of the PSD theory, originally proposed by [8], in order to characterize the performance sensitivity of manufacturing or technological engineering processes, such as injection molding.

State-of-the-art PSD is a promising theory, developed in the field of robotics to evaluate the sensitivity of articulated mechanisms with respect to their dimensional variability [16]. The PSD theory describes the sensitivity of mechanical systems, allowing the synthesis of tolerances on design variables and measuring changes in performance due to the unpredictable variation of key parameters [17-20].

The specialized method workflow is shown in Fig. 1. First a PSD based description of the sensitivity problem is proposed. Then Regression Analysis is applied to calculate analytical relations, i.e. Performance Functions, between the targeted process performances and the key parameters, i.e. the subset of the process parameters which have, according to the definition of KCs, the greatest influence on the process performances [12-15]. The Specialized Performance Sensitivity Distribution (Specialized PSD) method is finally developed and applied to perform the sensitivity analysis on manufacturing or technological processes.

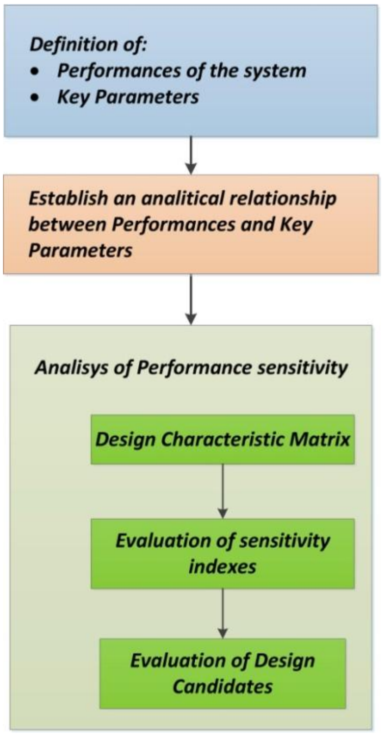


***Fig. 1 Specialized PSD method workflow.***

The paper is structured as follow: subsection 1.1 outlines the scientific foundation of the PSD method and describes its recent evolution.

Section 2 describes the mathematical specialization of the PSD theory for its adaptation to sensitivity evaluation in manufacturing and technological processes.

Section 3 deals with the application of the method to the analysis and characterization of the key parameters - shrinkage and flatness - for the injection molding process of a simple plastic specimen.

In Section 4 conclusions and future works are briefly drawn.

## 1.1 PSD theory evolution

The concept of sensitivity describes the relation between system performances and manufacturing tolerances, considered as internal system parameters.

Parkinson analyses the problem of determining optimal nominal dimensions of manufactured components subjected to dimensional tolerances [9]. He proposes a deterministic optimization process to calculate optimal nominal dimensions for assemblies, given a set of dimensional tolerances on the singular components and the overall system, and, backwards, to synthesize tolerances starting from the nominal dimensions of the parts.

Zhu and Ting introduce the theory of Performance Sensitivity Distribution and propose the analytical and geometrical description of PSD in the n-dimensional variation space, i.e. the space spanned by the parameter deviations [8]. Figure 2 shows the geometric representation of a 3-dimensional performance variation space, i.e. the space of variations of the parameters $\delta q_i$, described as a hyperellipsoid. Size, orientation and shape, stressed by specific sensitivity indexes, describe system performance depending on the deviation of the related parameters. Moreover, they introduce the concept of Design Characteristic Matrix as analytical representation of the performance variation in generic systems.

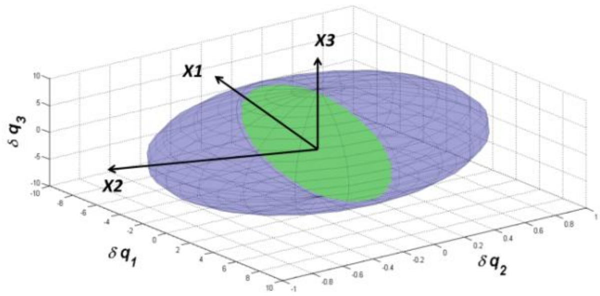


***Fig. 2 Hyperellipsoid in variation space (n=3).***

Caro et al. apply the PSD theory to develop a tolerance synthesis method for mechanisms, maximizing the hypervolume of the Tolerance Box, i.e. the volume surrounding hyperellipsoids. They propose alternative sensitivity indexes and compare their effectiveness [21]. The authors demonstrate also that the maximum singular value of the Design Characteristic Matrix can be used as an appropriate sensitivity index for mechanisms. Finally they present the synthesis of tolerances for a planar and a spatial manipulator.

Lu and Li integrate the PSD approach to the assessment of parameters in order to achieve stability and robustness on systems. They propose a method to manipulate the Characteristic Matrix through the placement of its eigenvalues [17].

In a more recent work, the same authors not only consider the relation between system performance and parameters but also address the problem of the uncertainty of the model itself. The approach consists of two separate optimization steps. The first minimizes the effect of the model uncertainties using the Matrix Perturbation theory applied to the Characteristic Matrix while the second minimizes the effect of parameter variations. Several case studies are finally presented [18].

Following the mathematical PSD approach, Al-Widyan and Angeles describe the relation between system performance and external environment parameters [16]. They particularly focus on the stochastic nature of such parameters and propose a theoretical framework, based on a probabilistic form of the Characteristic Matrix, for describing performances as functions of random variables normally distributed.

The PSD method is finally extended to consider also systems characterized by strong nonlinearity [19, 20].

From the literature review appears that the PSD theory and its variants have been usually applied to the characterization and analysis of mechanisms and engineering products. In order to extend such method to the characterization of processes, a different analytical description of the performance functions has to be proposed, specializing the PSD theory introduced by [8].

Regression Analysis provides the analytical tools needed to integrate the method and achieve the full characterization of processes in terms of their performance sensitivity.

Following such integrated approach it is possible to focus on the overall performance of processes and evaluate their performance sensitivity.

## 2 Specialized PSD method

This section provides the mathematical basics needed to specialize the PSD theory to perform the sensitivity analysis of manufacturing or technological processes.

In subsection 2.1. the definition of the sensitivity problem is proposed: Design Variables (DVs), Design Parameters (DPs) and Performance Functions (PFs) are firstly defined. The Design Characteristic Matrix is introduced and the geometrical description of the Feasible Space is briefly discussed.

In subsection 2.2. some topological criteria, based on the concept of hyperellipsoid, are proposed to quickly perform the sensitivity analysis.

In subsection 2.3 a regression model is proposed to describe the relation between PFs and DVs in a generic manufacturing or technological process.

### 2.1 Definition of the sensitivity problem

In sensitivity analysis key parameters can be distinguished in internal DVs, describing factors which can be taken under control and vary within known range, and external DPs, which cannot be controlled in working conditions.

The analytical functions used to described the relation between DPs, DVs and performances are defined as PFs [4].

DVs and DPs can be respectively described by the n-dimensional vector $\bar{\mathrm{q}} = [\mathrm{q}_1, \dots, \mathrm{q}_n]^{\mathrm{T}}$ and by the l-dimensional vector $\bar{\mathrm{p}} = [\mathrm{p}_1, \dots, \mathrm{p}_l]^{\mathrm{T}}$.

A Performance Function can be described by the m-dimensional vector $\bar{\mathrm{f}} = [\mathrm{f}_1, \dots, \mathrm{f}_m]^{\mathrm{T}}$.

System performances are related to the key parameters by the relation $\bar{\mathrm{f}} = \bar{\mathrm{f}}(\bar{\mathrm{q}}, \bar{\mathrm{p}})$.

The variation of performances $\Delta$, caused by the deviation of DVs and DPs, can be approximated by the linear expansion:

$$\boldsymbol{\Delta} = [\boldsymbol{J_q}\, \boldsymbol{J_p}][\overline{\boldsymbol{dq}^T} \quad \overline{\boldsymbol{dp}^T}]^{\boldsymbol{T}} = \boldsymbol{J}\, \boldsymbol{dX} \qquad (1)$$

where J represents the $m$ by $(p+q)$ Jacobian matrix of the system, evaluated for a particular set of DV and DP values, for instance the nominal values. It is composed by two parts: $\boldsymbol{J_q} = \boldsymbol{\partial \bar{f} / \partial \bar{q}}$ is the $m$ by $q$ Jacobian matrix of the DVs and $\boldsymbol{J_p} = \boldsymbol{\partial \bar{f} / \partial \bar{p}}$ is the $m$ by $p$ Jacobian of the DPs. The matrix J mathematically describes the sensitivity of the system and contains the variation of DVs ($\overline{\mathrm{dq}^{\mathrm{T}}}$) and DPs ($\overline{\mathrm{dp}^{\mathrm{T}}}$).

Equation (1) is valid under the condition that the variations of key parameters are less than 3%÷5% of the nominal value otherwise the Jacobian matrix of the system has to be modified [9].

The domain which contains all the possible variation of DPs and DVs is the variation space. Such space is spanned by the components of dX, supposed as independent.

According to the scientific literature, in first approximation variations of DPs are negligible because are generally associated to parameters which have a little influence on the final performances compared with DVs. A typical example of DP for an injection molding process is external humidity [21].

The norm of the vector $\Delta$ allows one to constrain the performances variation, according to the process targets:

$$\|\boldsymbol{\Delta}\|_2 = [\boldsymbol{J}\, \boldsymbol{dX}]^{\boldsymbol{T}}[\boldsymbol{J}\, \boldsymbol{dX}] = \sum_{\boldsymbol{i}=1}^{\boldsymbol{m}} \Delta_{\boldsymbol{i}}^2 \qquad (2)$$

In equation (2) the value of quadratic norm is the sum of the square values of the individual performance tolerances.

Let the Characteristic Design Matrix be $\mathrm{A} = \mathrm{J}^{\mathrm{T}}\mathrm{J}$, equation (2) becomes:

$$\|\boldsymbol{\Delta}\|_2 = [\, \boldsymbol{dX}\,]^{\boldsymbol{T}} \boldsymbol{A} [\boldsymbol{dX}] \qquad (2.1)$$

The equations (2) and (2.1) state that the Design Characteristic Matrix is semi-positive definite and has n orthonormal eigenvectors and n nonnegative eigenvalues. The number of positive eigenvalues is equal to its rank.

The eigenvectors of A define size, shape and orientation of a hyperellipsoid within a family depending by the scalar value $\|\Delta\|_2$ and the points on the hyperellipsoid surface are represented by the same value $\|\Delta\|_2$.

The lengths of the semiaxes are inversely proportional to eigenvalues, so the performances are less sensitive in the direction of the biggest eigenvalues and more sensitive in the direction of the smallest ones. When some eigenvalues assume a zero value, the hyperellipsoid degenerate to a cylindroid.

Summarizing, given a family of hyperellipsoids sharing the same values of PF, each member is described by a set of DVs.

Sensitivity analysis raises from the evaluation of sets of different DVs, also referred as Design Candidates.

Such evaluation needs to impose mathematical constraints on performances, for example by defining dimensional tolerances on lengths. Performance constraints, also called performance tolerances, are defined by engineers according to the design requirements.

Let $\Delta^*_i$ be a singular performance tolerance, the global constraint on performances has the following mathematical expression:

$$\|\Delta\|_2 \leq \sum_{\boldsymbol{i}=1}^{\boldsymbol{m}} \Delta^{*2}_{\boldsymbol{i}} = \hat{\Delta}_{\boldsymbol{r}}^2 \qquad (3)$$

where $\hat{\Delta}_{\boldsymbol{r}}^2$ is the sum of the squared individual performance tolerance and defines a bounded volume inside the hyperellipsoid, also called Feasible Space. The Feasible Space is that portion of the variation space where the gross system performance is acceptable.

When the Feasible Space is topologically described by a cylindroid it becomes unbounded and no sensitivity analysis can be performed for a certain direction. Then the cylindroid has to be re-conducted to a hyperellipsoid under the condition that the directions of the principal axes remain the same.

The solution of such problem is given in [8]; the algorithm (4) is presented to modify the Feasible Space through the adjustment of the eigenvalues of the Characteristic Design Matrix:

$$\hat{\boldsymbol{\lambda}}_{\boldsymbol{i}} = \boldsymbol{max}\left(\boldsymbol{\lambda_i}, \frac{\hat{\Delta}_r^2}{\boldsymbol{k}^2\, \|\boldsymbol{q}\|_2^2}\right) \qquad (4)$$

where $\lambda_i$ is the i-th eigenvalue of the Characteristic Design Matrix $\|\mathrm{q}\|_2^2 = \sum_{\mathrm{i}=0}^{\mathrm{n}} \mathrm{q_i}^2$ is the nominal value of DVs and k is a coefficient chosen in the range [0,03÷0,05] to satisfy the linearity requested by equation (1).

### 2.2 Topological criteria for sensitivity analysis

A topological criterion for performing the sensitivity analysis exploiting hyperellipsoids can be now defined.

Setting n=2 the hyperellipsoid becomes the ellipse $\xi\,(\mathrm{C})$, as shown in Figure 3.

The area contained by the ellipse represents the Feasible Space, while the rectangular area inscribed in the ellipse represents the variations $\delta x_1$, $\delta x_2$ which respect the given requirements on the performances (Tolerance Box).

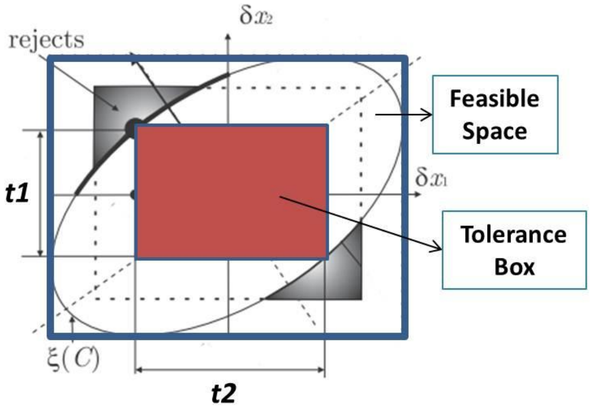


*Fig. 3 Hyperellipsoid in the variation space (n=2) [21].*

The robustness of the system is maximized when its sensitivity is minimized, so that the size of the Feasible Space is maximum and the Tolerance Box covers the most part of the Feasible Space as possible without overcoming the limits of the ellipse. The values taken outside the ellipse causes degradation of the final performances and must be rejected.

Considering a hyperellipsoid, the ratio between the Feasible Space and the hypervolume of the n-dimensional Tolerance Box depends on the orientation of the main axes. The orientation depends on the eigenvectors of the Design Characteristic Matrix.

### 2.3 PSD specialization

Sensitivity indexes can be introduced to evaluate size and orientation of the Feasible Space. The best Design Candidates are described by the following indexes, presented in order of importance:

- Minimum value for the maximum eigenvalues of the Design Characteristic Matrix ($\hat{\lambda}_N$);
- Maximum Feasible Space ($V_f$);
- Minimum ratio between the Feasible Space and the hypervolume of the n-dimensional Tolerance Box ($\beta_u$).

For complex systems, a Design Candidate (a set of DVs) may not simultaneously satisfy all these conditions, so different trade-offs are presented in the literature.

To apply the PSD theory an analytical model of the process has to be defined for the PF. Such model cannot be described by a linear relation. In fact, according to (2.1) the Design Characteristic Matrix must directly depend on the DVs.

A Regression Analysis technique is then adopted to specialize the PSD theory for the sensitivity analysis of processes. In particular, the PF for a process can be approximated and mathematically modelled by a fitted second-order polynomial regression model, which is called quadratic model, defined as:

$$\boldsymbol{f_i(\bar{q}) = \alpha_i + \overline{\beta_i}^T \bar{q} + \bar{q}^T Q_i \bar{q}} \qquad \boldsymbol{i=1,..,m} \qquad (5)$$

where the scalar $\alpha_i$, the vector $\overline{\boldsymbol{\beta_i}}$ and the matrix $\boldsymbol{Q_i}$ represent the regression coefficients.

The order of the model requires a number of $10 \cdot \mathrm{m}$ coefficients, 10 for each PF.

According to equation (5), it is possible to obtain the following expression for the Jacobian matrix:

$$\boldsymbol{J_i(\bar{q}) = \bar{\beta}_i + 2Q_i\bar{q}} \qquad (6)$$

The Jacobian Matrix of the process then becomes:

$$\boldsymbol{J(\bar{q})} = [\boldsymbol{J_1}, \boldsymbol{J_2}, \dots, \boldsymbol{J_m}]^T \qquad (7)$$

and the Design Characteristic Matrix for each DV can be calculated.

Sensitivity indexes are finally evaluated to perform the sensitivity analysis.

Sensitivity indexes depend on the particular point where they are evaluated. For n=2 or n= 3, the variation of the sensitivity indexes can be graphically described in the domain of DV by maps of colour, also called sensitivity maps. Such maps plot the values assumed by the sensitivity indexes within the domain of DVs and represent an effective tool to quickly evaluate the process sensitivity.

## 3 Injection Molding process

The present section addresses the sensitivity analysis of a process of injection molding for a simple rectangular plastic specimen. The goal is the selection of the Design Candidate which minimizes the process sensitivity, simulated trough the CAE software Moldex3D by CoreTech System Co., Ltd.

The rectangular specimen measures 60x30mm and has a thickness of 3mm. The material is polysulfone (PPSU), a high performance engineering thermoplastic polymer mainly used for automotive and biomedical applications [22]. The DVs are: Mold Temperature (MoT, $q_1$), Melt Temperature (MeT $q_2$) and Packing time (Pat, $q_3$) and are stored in the vector $\bar{q} \in \mathbb{R}^3$.

Each component belongs to a specific range, as reported in Table 1.

| *Design Variables* | *Lower limit* | *Upper limit* |
|---|---|---|
| *Mold Temperature [°C]* | 120 | 180 |
| *Melt Temperature [°C]* | 340 | 400 |
| *Packing time [s]* | 3 | 17 |

***Table 1 Specific ranges for nominal DV.***

The PFs are Shrinkage ($f_1$) and Flatness ($f_2$), which constitute the vector $\bar{f} \in \mathbb{R}^2$. Shrinkage data are directly provided by the CAE simulation while Flatness is calculated considering the displacement of some measure points with respect to their initial position, in according to [23].

The necessary data needed to build the response model are obtained applying a simulation experimental design. A Circumscribed Central Composite Design technique is adopted in order to fit the quadratic model for the PFs (Figure 3).

Table 3 shows the values of performances given by the process CAE simulation.

Regression Analysis is then performed to define the PFs for the injection molding process.

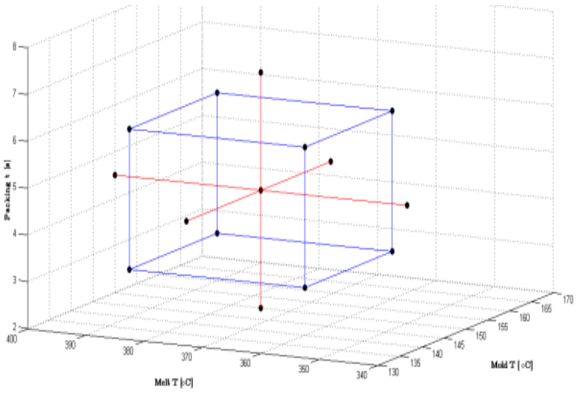

*Fig. 3 Simulated Points.*

The analysis of the coefficients in Table 2 leads to observe that each matrix Q isn't positive-definite, so a local optimum does not exist.

| | *Shrinkage (1stPF)* | *Flatness (2thPF)* |
|---|---|---|
| $\alpha_i$ | -5.4520 | -3.6325e-002 |
| $\bar{\beta}_i$ | [3.3646e-002, 1.1196e-002, 1.7624e-001] | [-2.0725e-005, 2.1511e-004, -6.8257e-004] |
| $Q_i$ | [-8.2089e-005, 5.6250e-006, -4.4792e-004; 5.6250e-006, -2.2674e-005, 3.7583e-004; -4.4792e-004, 3.7583e-004, -1.1366e-002] | [3.8687e-008, -4.1667e-008, 1.2500e-006; -4.1667e-008, -1.2572e-007, 2.7778e-007; 1.2500e-006, 2.7778e-007, -4.3804e-005] |

*Table 2 Coefficient for equation (5).*

Table 3 presents the results of the CAE simulation.

| Simulat. (n°) | MoT. [°C] | MeT. [°C] | Pat [s] | Flatness Tolerance [mm] | Shrinkage [%] |
|---|---|---|---|---|---|
| *1* | *140* | *360* | *3,0* | *0,0176* | *0,156* |
| *2* | *160* | *360* | *3,0* | *0,0170* | *0,366* |
| *3* | *140* | *390* | *3,0* | *0,0210* | *0,099* |
| *4* | *160* | *390* | *3,0* | *0,0204* | *0,315* |
| *5* | *140* | *360* | *6,0* | *0,0160* | *0,822* |
| *6* | *160* | *360* | *6,0* | *0,0156* | *0,977* |
| *7* | *140* | *390* | *6,0* | *0,0195* | *0,831* |
| *8* | *160* | *390* | *6,0* | *0,0190* | *0,995* |
| *9* | *133* | *375* | *4,5* | *0,0190* | *0,452* |
| *10* | *166* | *375* | *4,5* | *0,0179* | *0,754* |
| *11* | *150* | *350* | *4,5* | *0,0157* | *0,637* |
| *12* | *150* | *400* | *4,5* | *0,0210* | *0,595* |
| *13* | *150* | *375* | *1,9* | *0,0194* | *0,009* |
| *14* | *150* | *375* | *7,0* | *0,0169* | *1,106* |
| *15* | *150* | *375* | *4,5* | *0,0184* | *0,613* |

*Table 3 Simulation results.*

Response Surfaces (RSs) are then calculated at different temperatures for shrinkage (from Fig. 4a to Fig. 4d) and flatness (from Fig. 5a to Fig. 5d).

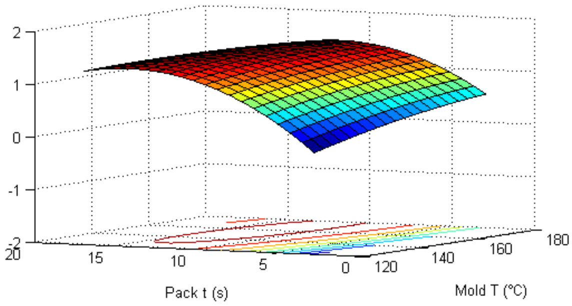


*Fig. 4(a) Shrinkage RS for a MeT of 340,0°C.*

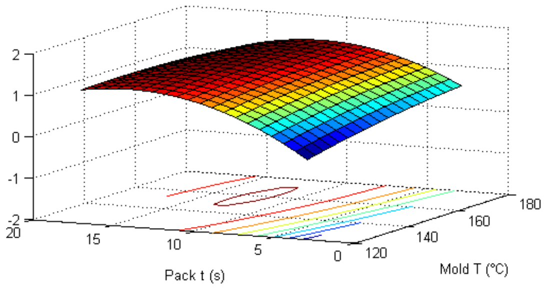


*Fig. 4(b) Shrinkage RS for a MeT of 333,4 °C.*

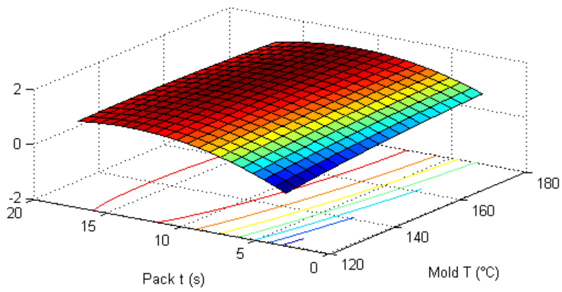


*Fig. 4(c) Shrinkage RS for a MeT of 327,0 °C.*

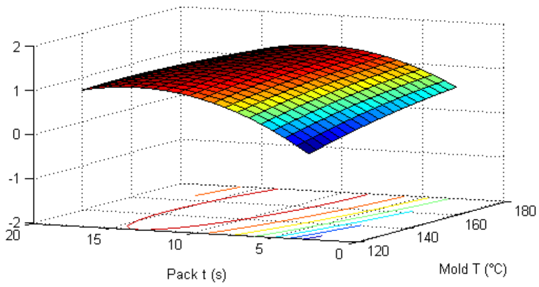


*Fig. 4(d) Shrinkage RS for a MeT of 320,0 °C.*

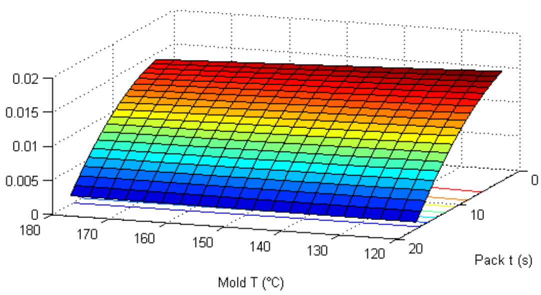


*Fig. 5(a) Flatness RS for a MeT of 340,0°C.*

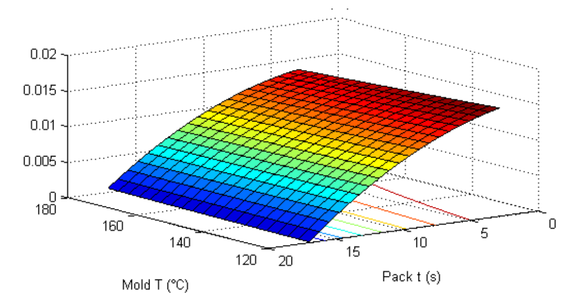


*Fig. 5(b) Flatness RS for a MeT of 333,4 °C.*

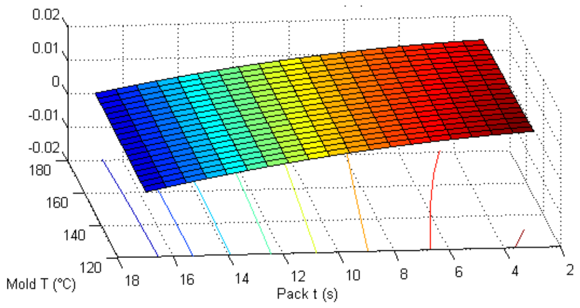


*Fig. 5(c) Flatness RS for a MeT of 326,7 °C.*

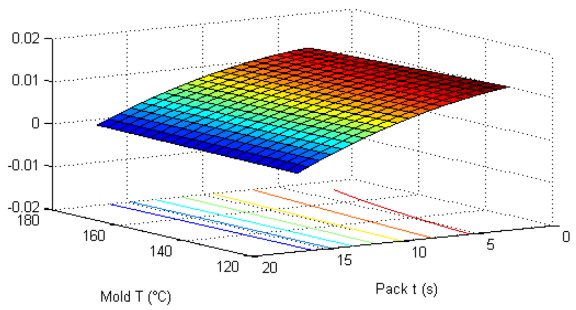


*Fig. 5(d) Flatness RS for a MeT of 320,0 °C.*

The analysis of sensitivity starts with the assumption that the coefficient k in equation (4) is equal to 0,035.

Equation (3) gives the global tolerance $\hat{\Delta}_r$ equal to 0,502.

Table 4 shows the considered Design Candidates, randomly chosen according to the material data sheet.

Sensitivity indexes are finally calculated and listed in Table 5, where the number of significant digits aims to discriminate the adequacy of the candidates.

The Design Candidate n. 12 presents the best trade-off with respect to the sensitivity indexes and it is chosen as optimum.

The sensitivity indexes are finally plotted and shown in Fig. 6 and Fig. 7 in form of sensitivity maps.

| *Design Candidates N°* | *MoT. [°C]* | *MeT [°C]* | *Pat. [s]* |
|---|---|---|---|
| 1 | 170,7 | 379,3 | 9,1 |
| 2 | 175,2 | 342,1 | 8,3 |
| 3 | 136,3 | 390,9 | 13,7 |
| 4 | 175,6 | 396,0 | 14,1 |
| 5 | 161,6 | 380,7 | 5,6 |
| 6 | 134,8 | 385,4 | 9,8 |
| 7 | 143,9 | 384,5 | 9,2 |
| 8 | 157,3 | 363,5 | 12,0 |
| 9 | 177,8 | 379,3 | 12,9 |
| 10 | 178,2 | 350,2 | 13,5 |
| 11 | 137,8 | 382,3 | 6,8 |
| 12 | 178,5 | 341,9 | 12,5 |
| 13 | 177,8 | 356,6 | 12,1 |
| 14 | 154,2 | 342,7 | 5,2 |
| 15 | 170,0 | 345,8 | 4,6 |
| 16 | 137,0 | 389,4 | 9,9 |
| 17 | 151,0 | 381,6 | 16,4 |
| 18 | 175,7 | 359,0 | 7,7 |
| 19 | 169,6 | 397,0 | 11,1 |
| 20 | 177,9 | 342,0 | 6,1 |

*Table 4 Design Candidates.*

| Design Candidates (N°) | Vf | $\beta_u$ | Max $\hat{\lambda}_N$ |
|---|---|---|---|
| 1 | 2298,0891 | 0,5184147 | 0,0316246 |
| 2 | 12873,8100 | 0,5239134 | 0,0011945 |
| 3 | 2797,7183 | 0,5174935 | 0,0230918 |
| 4 | 5914,6129 | 0,5217049 | 0,0052600 |
| 5 | 2454,6346 | 0,5190484 | 0,0383973 |
| 6 | 11424,9670 | 0,5239286 | 0,0012935 |
| 7 | 2580,7014 | 0,5164247 | 0,0300025 |
| 8 | 6429,4852 | 0,5193678 | 0,0045012 |
| 9 | 9982,2322 | 0,5238268 | 0,0015152 |
| 10 | 11550,4190 | 0,5239711 | 0,0012841 |
| 11 | 5490,5886 | 0,5218877 | 0,0048566 |
| 12 | 1682,6069 | 0,5072925 | 0,0448476 |
| 13 | 2668,4225 | 0,5204475 | 0,0261810 |
| 14 | 12592,5930 | 0,5238637 | 0,0012122 |
| 15 | 2433,4845 | 0,5192117 | 0,0396856 |
| 16 | 9704,4996 | 0,5239062 | 0,0014421 |
| 17 | 4429,9771 | 0,5175455 | 0,0086332 |
| 18 | 6642,1434 | 0,5176788 | 0,0036745 |
| 19 | 1873,6642 | 0,5149459 | 0,0381646 |
| 20 | 3939,9591 | 0,5194445 | 0,0100861 |

*Table 5 Sensitivity indexes.*

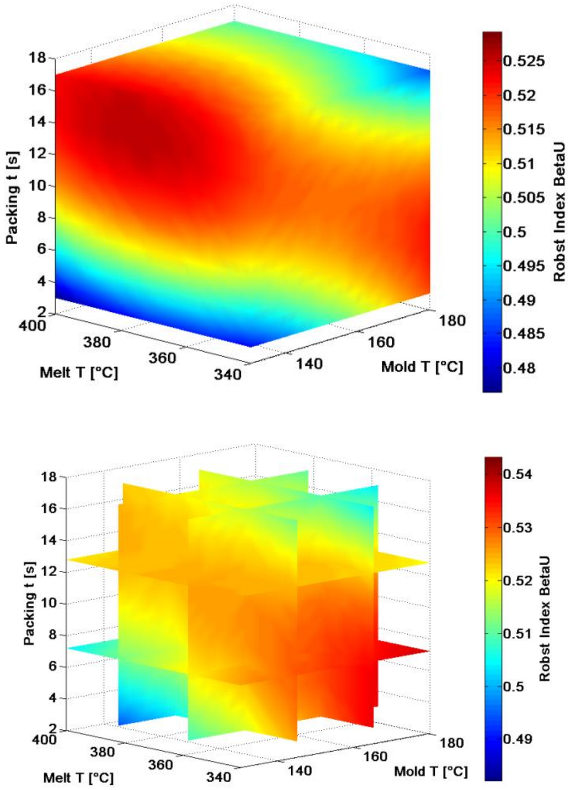


*Fig.6 Sensitivity map for index $\beta_u$.*

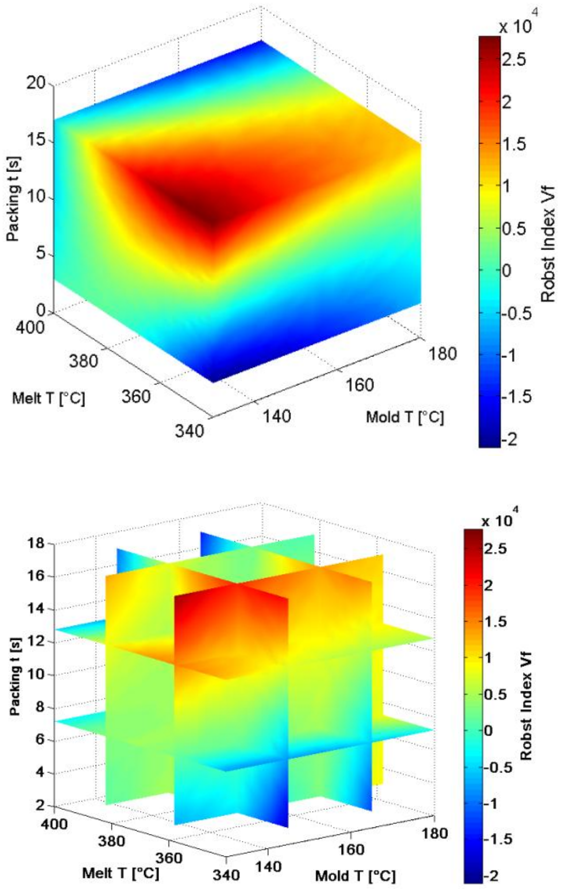


*Fig.7 Sensitivity map for index $V_f$.*

## 4 Discussion and Conclusion

Robust design techniques are very effective in product and process optimization but are not suited to perform sensitivity analysis.

The PSD theory was primarily developed to provide analytical and geometric description of performance sensitivity for mechanisms, especially in the field of robotics, but the theory presents the potentiality to be extended to other engineering fields.

The paper presents a specialization of the PSD theory in order to perform the sensitivity characterization of manufacturing and technological processes and investigate the variation space of DVs. In particular, actual PSD limitations were overcome through the formulation of a suitable analytical expression for the PFs, based on the adoption of Regression Analysis.

The description of performances in terms of process variables represents the key criterion for the applicability of the method presented, so its effectiveness depends on the accuracy of the regression model itself. At the present a second-order regression model is adopted but further studies are investigating the use of different models, in order to consider also non-quadratic effects.

The methods involves also the calculation of sensitivity indexes and proposes the plot, for n=2 or 3, of sensitivity maps of colour created to give an effective outlook of the process sensitivity directly in the 3D space of DVs and not in the space of their variations i.e. in the Feasible Space, as foreseen by the PSD theory. The integration of sensitivity maps in CAE software could be very useful to give the engineering designer an effective tool for integrated engineering design, especially in the early design stage of product and process development and is under investigation.

A simple case study, dealing with injection molding of a rectangular high performance engineering plastic specimen, is finally presented to validate the Specialized PSD method and find the Design Candidate that minimizes the sensitivity for shrinkage and flatness.

## Acknowledgements

The authors want to acknowledge Dr. F. Gherardini for his technical contribution and like to remember that the present work has been supported by the “Interdepartmental Centre for Applied Research and Services in the Field of Advanced Mechanics and Engine Design - INTERMECH-MO.RE.”, thanks to the European funds POR FESR 2007-2013 of the Emilia-Romagna region.

## References

[1] C. F. Wu Jeff and M. Hamada. Experiments: Planning, Analysis, and Parameter Design Optimization. Wiley, 2002.

[2] S. H. Park. Robust Design and Analysis for Quality Engineering. Chapman & Hall ,1996.

[3] Boeing Commercial Airplane Group, Key characteristics: the first step to advance quality, Report #D6-55596 TN, USA, 1992.

[4] General Motors Corporation, Key characteristics Designation System: Reference Manual, GM 1805 QN, 1996.

[5] A.C. Thornton, A mathematical framework for the key characteristic process, Research in engineering design, 11, 1999, pp.145-157.

[6] J.Y Dantan, A. Hassan, A. Etienne, A. Siadat, P. Martin, *Information modelling for Variation Management during the Product and Manufacturing Process Design*,

International Journal on Interactive Design and Manufacturing, Ed. Springer, Vol. 2, n°2, pp 107-118, 2008.
[7] D.C. Montgomery. Progettazione e analisi degli esperimenti. McGraw- Hill, 2005.
[8] J.M. Zhu and K.L. Ting. *Performance Distribution Analysis and Robust Design*. ASME Journal of Mechanical Design,123, 1, 2001, pp 11–17.
[9] D. B. Parkinson. *The application of a Robust Design Method to Tolerancing.* ASME Journal of Mechanical Design, 122, 2000, pp 149–154.
[10] W. Chen, J. Allen, K.L. Tsui and F. Mistree. *A Procedure for Robust Design: Minimizing Variations Caused by Noise Factors and Control Factors*. ASME Journal of Mechanical Design, 118 (4), 1996, pp 478–493.
[11] M.-S. Huang and T.-Y. Lin. *An innovative regression model-based searching method for setting the robust injection molding parameters*. Journal of Materials Processing Technology, 198, 3 ,2008, pp 436-444.
[12] H. Oktem, et al. *Application of Taguchi optimization technique in determining plastic injection molding process parameters for a thin-shell part.* Materials & Design 28, 4, 2007, pp 1271-1278.
[13] W.-C. Chen, G.-L. Fu, P.H. Tai, and W.J. Den *Process parameter optimization for MIMO plastic injectic molding via soft computing*. Expert Systems wi Applications, 36, 2, 2009, pp 114-1122.
[14] . Altan. *Reducing shrinkage in injection moldings via the Taguchi*, ANOVA and neural network methods. Materials & Design , 31, 1, 2009, pp 599-604 .
[15] S. Datta, G. Nandi and A. Bandyopadhyay. *Application of entropy measurement technique in grey based Taguchi method for solution of correlated multiple response optimization problems: A case study in welding*. Journal of Manufacturing Systems, 28, 2-3, 2009, pp 55-63.
[16] K. Al-Widyan and J. Angeles. *A Model-Based Formulation of Robust Design.* ASME Journal of Mechanical Design, 127,3, 2005, pp 388–396.
[17] X. Lu and H. Li. *Stability Based Robust Eigenvalue Design for Tolerance.* ASME Journal of Mechanical Design, 131, 8, 2009, 081007 (7 pp).
[18] X. Lu and H. Li. *Perturbation Theory Based Robust Design Under Model Uncertainty.* ASME Journal of Mechanical Design, 131, 11, 2009, 111006 (9 pp).
[19] X. Lu and H. Li and P. Chen. *Variable Sensitivity-Based Deterministic Robust Design for Nonlinear System.* ASME Journal of Mechanical Design, 132, 6, 2010, 064502 (7 pp).
[20] M.-S Huang and T.-Y. Lin. Simulation of a regression-model and PCA based searching method developed for setting the robust injection molding parameters of multi-quality characteristics. International Journal of Heat and Mass Transfer, 51, 25-26, 2008, pp. 5828–5837.
[21] S. Caro, F. Bennis and P. Wenger. *Tolerance Synthesis of Mechanisms: A Robust Design Approach.* ASME Journal of Mechanical Design, 127 ,1, 2005, pp. 86–94.
[22] BASFWebsite. http://www.plasticsportal.net/wa/EU~pl_PL/Catalog/ePlastics/pi/BASF/prodline/ultrason accessed 15 Jan 2011.
[23] ISO/TS 12781-1:2003, Geometrical Product Specifications (GPS) -- Flatness -- Part 1: Vocabulary and parameters of flatness.