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Parameter combinatorial diagram. Application to the study and analysis of Hydrological Simulation Models.

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Abstract

In this paper we define Parameter Combinatorial Diagram as the joint graphical representation of all box plots related to the adjustment between real and simulated data, by setting and / or changing the parameters of the simulation model. To do this, first we start with a box plot representing the values of an objective adjustment function, achieving these results when varying all the parameters of the simulation model. Then we draw the box plot when setting all the parameters of the model, for example using the median or average. Later we get all the box plots when carrying out simulations combining fixed or variable values of the model parameters. Finally all box plots obtained are represented neatly in a single graph. Besides being able to represent the box plots associated with an objective function it is possible to represent the diagrams associated with the variation of each parameter. It is intended that the new Parameter Combinatorial Diagram is used to examine and analyze simulation models useful in practice. This paper presents combinatorial diagrams of different examples of application as in the case of hydrological models of one, two, three, and five parameters.

1 Introduction

The tables represent information in outline and are ready for subsequent calculations. The graphics convey that information in a more expressive way, they will allow, with a single glance, to understand that we are talking about, to see its most important features, even to draw conclusions about the behaviour of the sample the study is being conducted [1]. It is known that when numerical data are available, and before tackling complex analysis or study, a first step is to present this information so that it can be viewed in a more systematic and summarised way. The most common graphics used in the analysis of results are: bar chart, histogram, frequency polygon, pie chart, box plot [2-3] and others. The application of simulation models involves delivering measurable results of that application or experiment. The clarity of the presentation is of vital importance for the understanding of the results and their interpretation. At the time of representing the results of a simulation analysis in an appropriate manner, there are several publications that we can consult [4-7]. Although the presentation of numerical data is usually done using tables, sometimes a chart or graph can help to represent our data in a more efficient way.

This article will address the graphic representation of the results of the application of simulation models using Parameter Combinatorial Diagram, noting its usefulness in the process of data analysis and presentation.

2 Graphical representation of simulation results

Simulation is the process of designing a model of a real system and carry out experiments with it, in order to understand their behaviour or to evaluate new strategies for its operation [8]. The real system model can be theoretical, conceptual or systemic and must contain the elements necessary for the simulation (scenario or set of working hypotheses) [9]. The steps to perform a simulation study can be specified in:

— System definition. Is to study the context of the problem, identify the objectives of the project, specify the indices measuring the effectiveness of the system, specify the specific objectives when defining the system to be modelled.

— Model formulation. Once precisely defined results to be obtained from the study, we define and build the model which will yield the desired results. In the formulation of the model it is necessary to define all variables and parameters that are part of it, their logical relationships and flow charts describing in full the model.

— Data collection. It is important to define clearly and precisely the data that the model will be required to produce the desired results.

— Implementation of the model in the computer. With the model defined, the next step is to write a computer program to process and obtain results for comparison.

— Verification and validation. The verification and validation process is about checking that the simulated model meets the design requirements for which it was developed. Throughout this stage we check the differences between the operation of the simulator and the real system through the quantification of an objective function, which is the one that we want to optimize, ie, maximize or minimize.

— Experimentation. Experimentation with the model is done after it has been validated. The experiment is to generate the desired data and to carry out a sensitivity analysis of the levels required.

— Interpretation. At this stage of the study, the results shown by the simulation are interpreted and based on this we make a decision. The clarity of the representation of the results is of vital importance for the understanding and interpretation of same.

The Parameter Combinatorial Diagram is defined as the joint graphical representation of all box plots related to the adjustment between real and simulated data, when setting and / or change the parameters of the simulation model. These are the various stages to specify the combinatorial diagram of n parameters associated with a simulation model:

— Data selection. We define the data that the simulation model is going to require to produce the desired results.

— Definition of the objective function. We check the differences between the operation of the simulated system and the real system through the quantification of an objective function, which is the one that we want to optimize, ie, maximize or minimize.

— Application of model simulation varying all parameters. We represent the box plot obtained when varying all the parameters of the simulation model.

— Choosing fixed values of the parameters. From the values represented when varying all parameters, we choose a statistical value, such as median or mean, to set the parameters of the model.

— Application of the simulation model by setting all parameters. We draw the box plot obtained by the simulation by fixing all model parameters.

— Application of the simulation model setting i the parameters. We obtain the $n!/i!(n-i)!$ box plots when carrying out simulations combining the i fixed values and the $n-i$ varying values of model parameters.

— Parameter Combinatorial Diagram representation. The final phase is represented neatly in a single graph (fig. 1) all box plots obtained.

The number of box plots that make up the Parameter Combinatorial Diagram is 2^n, which corresponds to the sum of all possible combinations that exist when fixing or varying the n parameters of the simulation model.

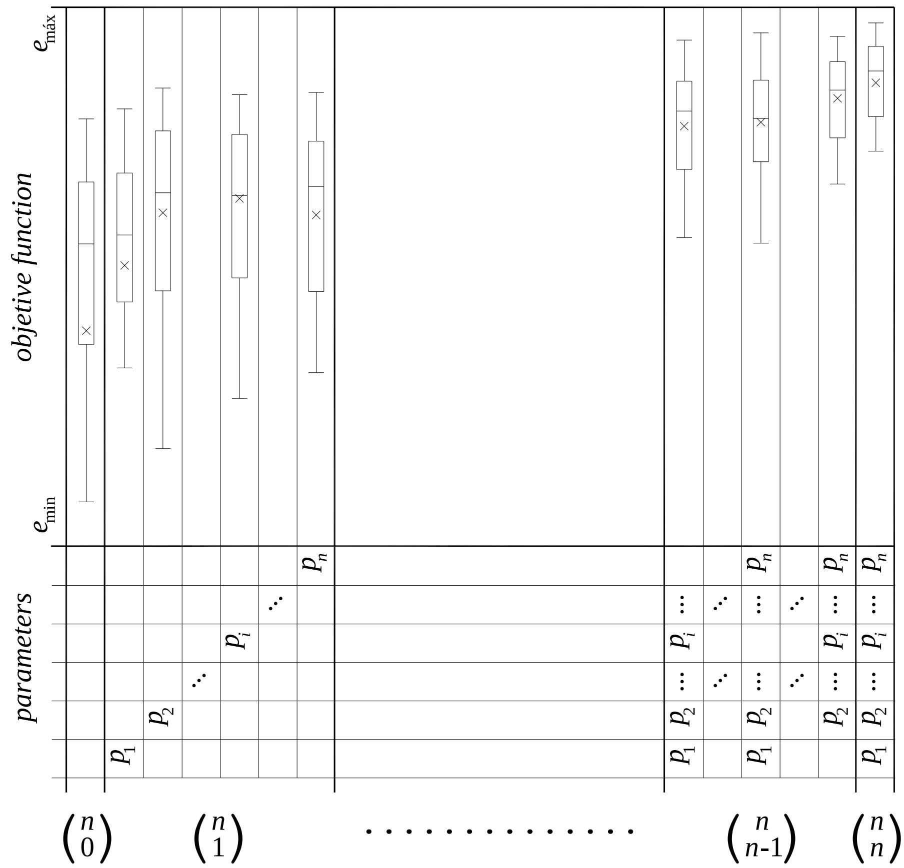


Fig. 1 Parameter Combinatorial Diagram of an objective function

Besides being able to represent the box plots associated to the values of an objective function that we want to optimize in the Parameter Combinatorial Diagram, it is also possible to represent the box plots associated to the values of each of the parameters. In this case we can simplify the representation (fig. 2).

The number of box plots, in the case of representing the values of a parameter, is 2^{n-1}, which corresponds to the

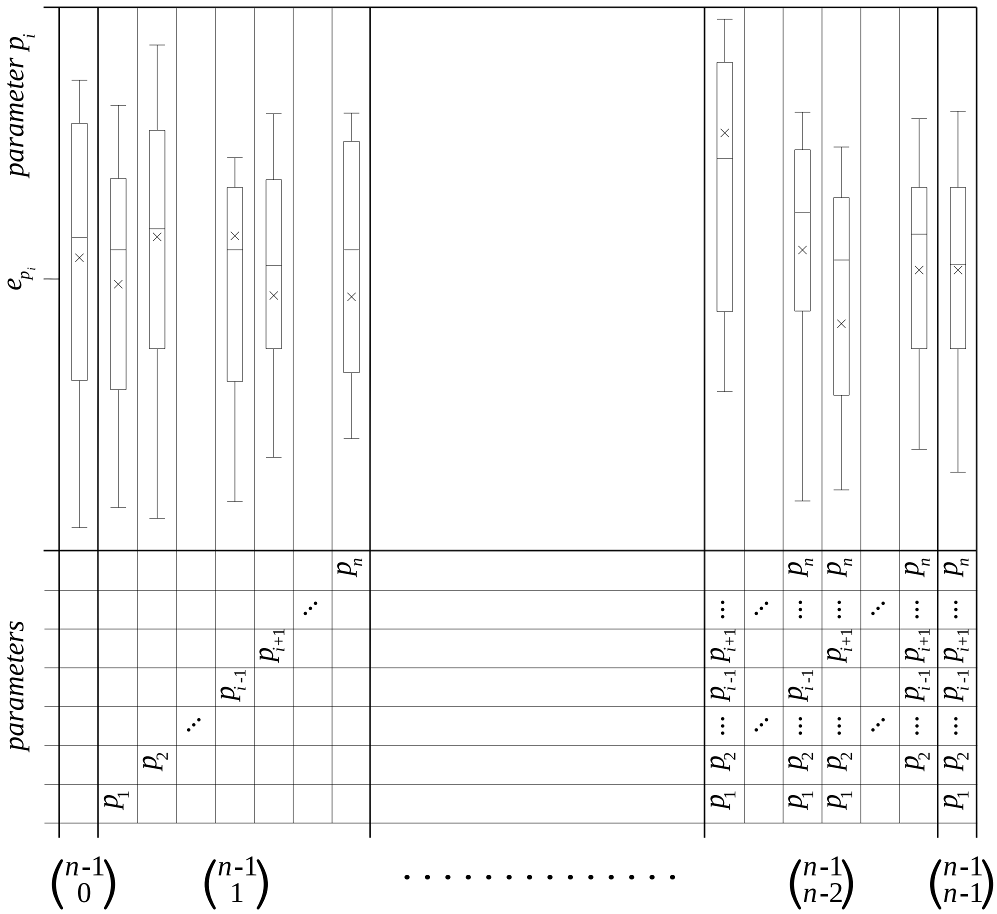


Fig. 2 Parameter Combinatorial Diagram of the value of a parameter

sum of all possible combinations that exist when fixing or varying the n–1 remaining parameters of the simulation model.

3 Parameter Combinatorial Diagram in hydrological models

In this paper we presents combinatorial diagrams of different examples of application as in the case of hydrological models of one, two, three, and five parameters. This section has been structured in steps in order to accurately describe the different Parameter Combinatorial Diagrams.

Description of the watershed subject to study

The hydrological model simulations have been conducted in the watershed of Aixola [10] located in northern Spain in the province of Gipuzkoa. This watershed has a gauging station as part of the hydrometeorological network of Gipuzkoa Provincial Council which records the flow of the river every ten minutes. The precipitation data are also recorded every ten minutes by a rain gauge located at the gauging station itself. Aixola watershed is located on the western boundary of the province of Gipuzkoa and is mostly used with forest (> 85% surface) it has an area of 4,70 km², with extreme levels of 315 and 740 meters, an average slope of 44,25 % and an annual average rainfall of 1600 mm.

Description of events

From the recorded data we have selected a series of twenty events that could adapt to the hypothesis of unit hydrograph technique, applicable to hydrological models of one, two and three parameters subsequently submitted. The base flow has been extracted from each of these events and we have obtained effective precipitation hyetograph. The base flow extraction was performed using a recursive filter, namely the one proposed by Eckhardt [11] of two parameters. After removing the base flow we have obtained the effective precipitation hyetograph using the curve number method developed by the Soil Conservation Service [12] adjusting the observed direct runoff volume and the beginning of direct runoff hydrograph observed. In addition we have selected a series of thirty-seven events that could adapt to the application scenarios of the hydrological model of five parameters that it presents. The main features of each of these selected events are on the doctoral thesis titled "*Development of a rainfall-runoff simulation in humid areas. Implementation and evaluation in headwater catchments located in Gipuzkoa*" [13].

Definition of the objective function

In the application of hydrological models, the goodness of the adjustment of simulated hydrographs was made regarding the efficiency defined by Nash and Sutcliffe [14]. Thus the objective function is:

$$E = 1 - \sum_{j=1}^{m} \left[Q_{ob,j} - Q_{si,j} \right]^2 \Big/ \sum_{j=1}^{m} \left[Q_{ob,j} - \overline{Q_{ob}} \right]^2 \qquad (1)$$

where: $Q_{ob,j}$ is the flow is observed at the time j, $Q_{si,j}$ is the flow simulated at the time j and $\overline{Q_{ob}}$ is the average flow observed.

Combinatorial diagrams associated with Reservoir Geomorphological Instantaneous Unit Hydrograph

The hydrological model called the Reservoir Geomorphological Instantaneous Unit Hydrograph, RGIUH, has the following expression [15]:

$$h(t) = \frac{\alpha e^{-\alpha \frac{t}{\tau}}}{\tau} \sum_{i=1}^{n} \left[\frac{A_i}{(i-1)!} \left[\alpha \frac{t}{\tau} \right]^{i-1} \right] \qquad (2)$$

To determine the value geomorphological value $\alpha = \sum_{i=1}^{n} [i A_i] \Big/ \sum_{i=1}^{n} [A_i] = 5,25$ provided by the area relations, the watershed has been divided into sub-watersheds from the permanent drainage system represented in the mapping 1:5000. The only uncertain parameter τ represents the centre of gravity of the hydrograph. To obtain the direct runoff hydrograph simulated from the instantaneous unit hydrograph we apply the convolution equation to each effective precipitation hyetograph.

We present in fig. 3 the Parameter Combinatorial Diagrams associated the to objective function E and to the parameter value τ, obtained by simulating events.

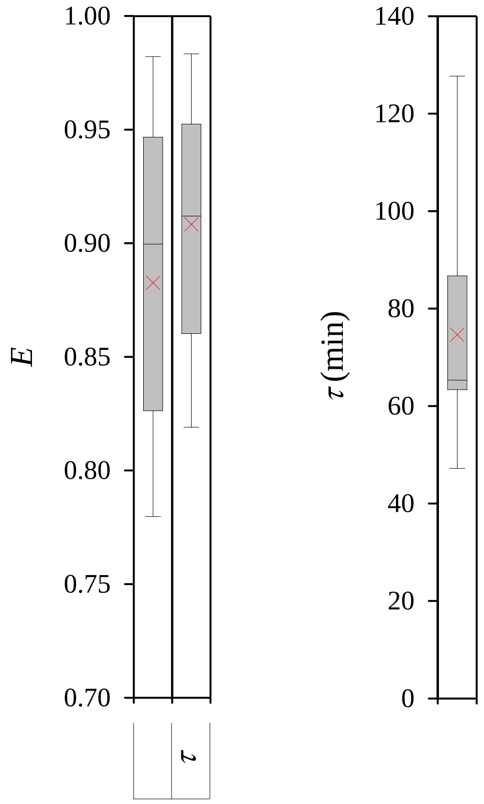


Fig. 3 Parameter Combinatorial Diagram of RGIUH

In this case the combinatorial diagram expresses a relationship with a single variable. You can analyze the strengths arising from the model when fixing or varying its only parameter.

Combinatorial diagrams associated with Instantaneous Unit Hydrograph represented by the Beta function of two parameters

The hydrological model called the Instantaneous Unit Hydrograph represented by the statistical function of two parameters Beta, β_τ, has the following expression [13]:

$$\beta_\tau(t) = \frac{\Gamma\left(3[1+\alpha_p]/[1-\alpha_p]\right) \tau^{p_\tau+1} t^{p_\tau-1}}{\Gamma\left([1+2\alpha_p]/[1-\alpha_p]\right) \Gamma\left([2+\alpha_p]/[1-\alpha_p]\right) [\tau+t]^{2p_\tau+1}} \qquad (3)$$

In this model the two parameters of the function β_τ are: α_p and τ. We present in fig. 4 Parameter Combinatorial Diagrams associated with the objective function E and the values of the parameters α_p and τ, obtained by simulating the events.

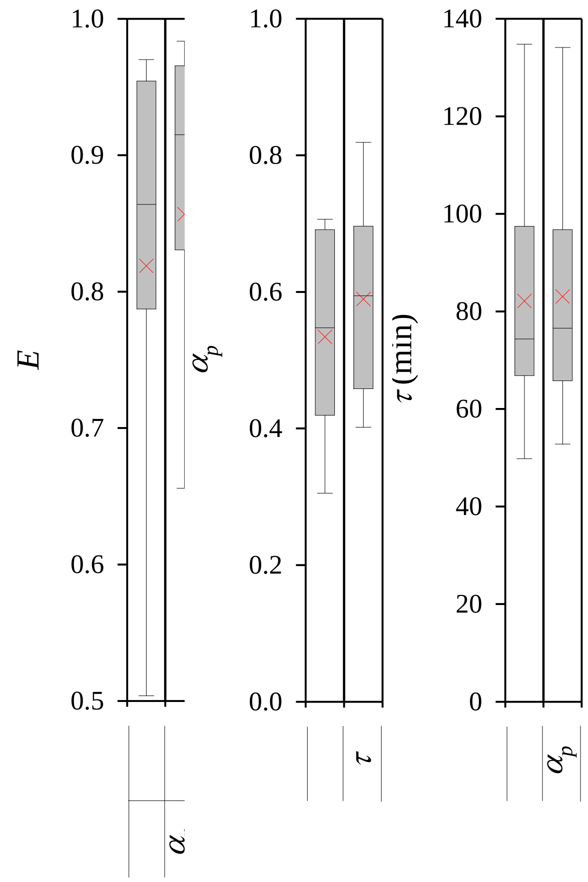


Fig. 4 Beta function Combinatorial Diagrams two parameters

In this case the combinatorial diagram expresses relationships with two variables. These relationships can be represented by a surface drawing.

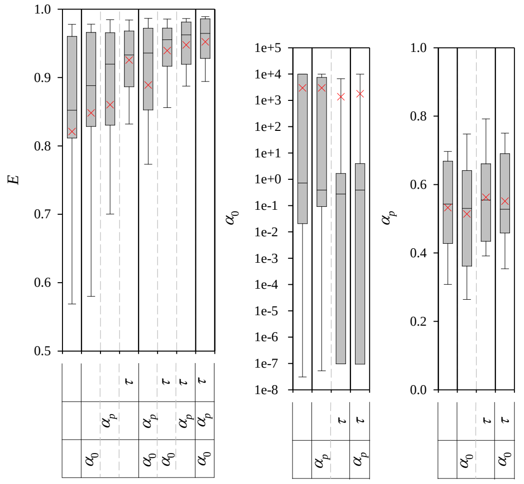
Combinatorial diagrams associated with the Instantaneous Unit Hydrograph represented by the Beta function of three parameters

The hydrological model called the Instantaneous Unit Hydrograph represented by the Beta statistical function of three parameters, β, has the following expression [13]:

$$\beta(t) = \frac{\Gamma\left([\alpha_0+\alpha_p][\alpha_0+2]/[\alpha_0[1-\alpha_p]]\right)[\alpha_0\tau]^q t^{p-1}}{\Gamma\left([\alpha_0+2\alpha_p]/[\alpha_0[1-\alpha_p]]\right)\Gamma\left([1+\alpha_0+\alpha_p]/[1-\alpha_p]\right)[\alpha_0\tau+t]^{p+q}} \qquad (4)$$

this model the three parameters of the function β are α_0, α_p and τ.

We present in fig. 5 the Parameter Combinatorial Diagrams associated with the objective function E and the parameter values α_0, α_p and τ, obtained by simulating the events.



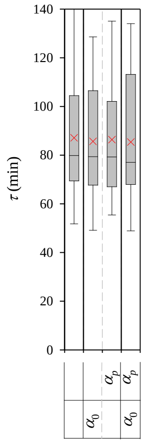


Fig. 5 Beta function Combinatorial Diagrams of three parameters

The combinatorial diagram of three parameters expresses, among other things, a volumetric drawing. It clearly displays the different influence of the parameters with respect to the adjustment made.

Combinatorial diagram associated with the Reservoir Rainfall-Runoff Geomorphological Model of five parameters

In fig. 6 we represent the operation of the hydrological model called rainfall-runoff geomorphological reservoirs

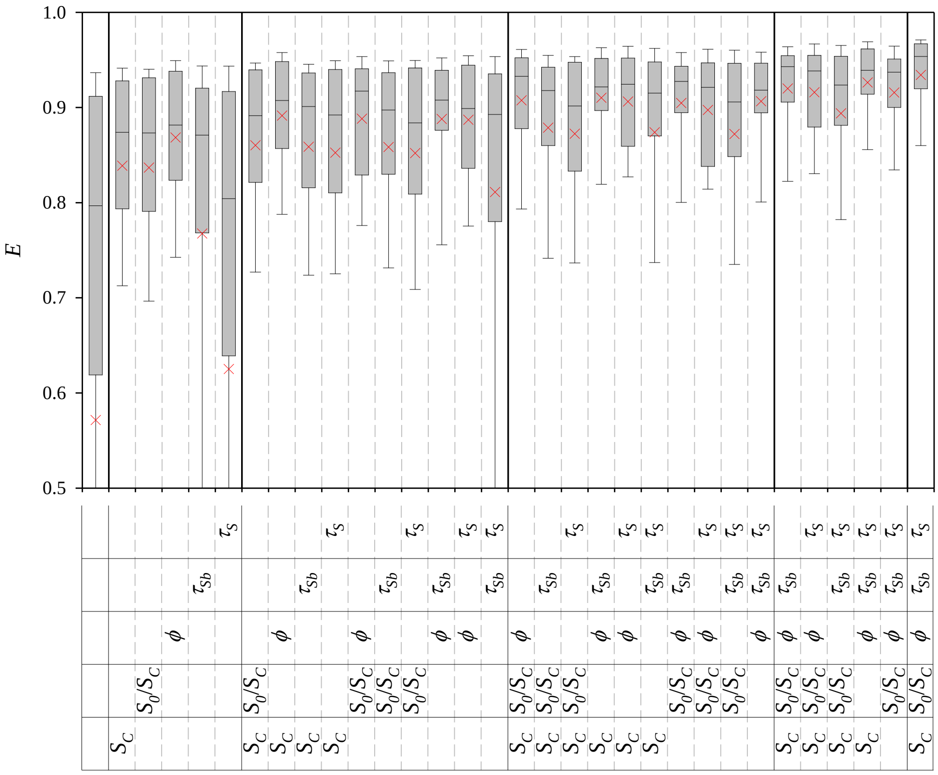


Fig. 7 Parameter Combinatorial Diagram of model R^3GeM of objective function E

model, R^3GeM [13].

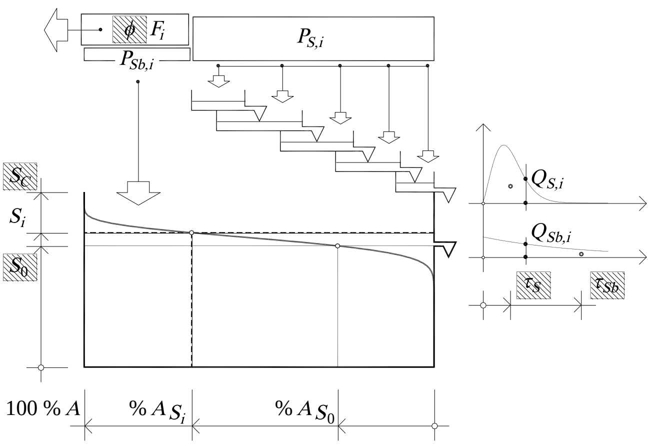


Fig. 6 Rainfall-runoff geomorphological reservoirs model,

$\mathrm{R^3GeM}$

In this model the five parameters are S_C, S_0/S_C, ϕ, τ_{Sb} and τ_S. We present in fig. 7 the Parameter Combinatorial Diagram associated with the objective function E obtained by simulating the events.

Analyzing the combinatorial diagram shown in the figure above we can deduce that varying only the parameter ϕ an efficiency E is acquired close to that obtained by varying all parameters. This means that said parameter is the most suited to calibrate the model. In turn, we can see that setting only the parameter S_0/S_C we can achieve practically the same efficiency than varying it. This means that you can replace this parameter by a fixed value.

We present in fig. 8 the Parameter Combinatorial

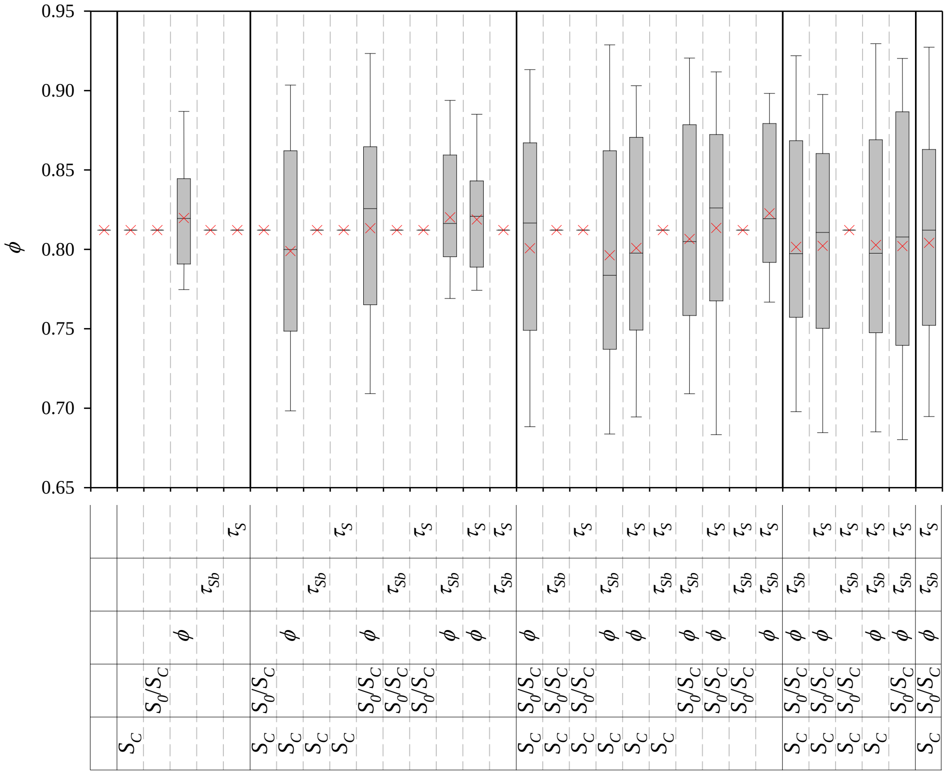


Fig. 8 Parameter Combinatorial Diagram of model $\mathrm{R^3GeM}$ ***of parameter*** ϕ

Diagram associated to parameter ϕ obtained by simulating the events.

You can see that when only the parameter ϕ is varied, this has less deviation than in the case of not setting any parameters. The simplified Parameter Combinatorial Diagram has not been used in fig. 8, with the aim of better understanding the organization of these diagrams.

4 Conclusions

The Parameter Combinatorial Diagram is an orderly and adequate representation of all box plots related to the adjustment between real and simulated data, by setting and / or change the parameters of the simulation model. We have been able to represent the combinatorial diagrams related to an objective function and the variation of each parameter in order to analyze more deeply the relationship between the parameters of a simulation model. The use of combinatorial diagrams associated with models of one or two parameters, like the models RGIUH and β_τ, is analogous, in terms of information provided, to the representations of lines and surfaces. The combinatorial diagrams of three or more parameters, like the models β and $\mathrm{R^3GeM}$, are best for displaying the adjustments made. The use of the Parameter Combinatorial Diagram is advisable to study and analyze simulation models.

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