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Main axonometric system related views as tilt of the coordinate planes

Lázaro Gimena [(a)], *Pedro Gonzaga* [(a)], *Faustino N. Gimena* [(a)]

[(a)] Department of Projects Engineering. Public University of Navarra

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Corresponding author:
Faustino N. Gimena
Tel.: +34 948 169 225
e-mail: faustino@unavarra.es
Address: Campus de Arrosadía, s/n. 31006 Pamplona. Spain

Abstract

In this communication we start both from a trirectangular trihedral defined by the three coordinate planes and a fourth plane, called the chart or projection, which contains the vertex of the trihedral above. Later we define the main related views or trihedral views as those produced by the tilt of the three coordinate planes to the outside of their trihedral on the chart plane. For each projection plane there are some unique main related views, regardless of the projection direction. In turn, from the trihedral views we can obtain the axonometric perspective from which they come. These properties can simplify some graphic constructions of the axonometric system, for example, determine the projections of the axes from the main related views or determine the axonometric scales from the axes. It is considered that this new definition of trihedral views allows a simplification as to the understanding of the systems of representation and is intended to be suitable for teaching purposes.

1 Introduction

At present knowledge of the axonometric perspective is perfectly coded and assumed, but this has not happened along the history and it has taken many centuries to achieve the complete representation formulation system that concerns us [1]. Axonometric perspective is scientifically systematised since the mathematical intervention. In 1820 W. Farish in his work, *Treatise on isometrical perspective*, he presents a new method of projection that is in fact an isometric orthogonal axonometry. In 1844 L. J. Weisbach in his work *Die monodimetrische und axonometrische. Projections methode*, provides the mathematical basis of orthogonal axonometric perspective. K. Pohlke states the theorem of the oblique axonometric perspective in his work *Darstellende Geometrie* (1860). In the document *Orthogonale Axonometrie* of 1905 Schesller systematises the orthogonal axonometric perspective. With this system of representation it was clearly defined from an operational point of view, with its most frequent use in scientific and technical fields. At present, there is little research on the axonometric perspective [2-7]. In this paper we extend the knowledge of the axonometric perspective and we define Main Related Views as those produced by the tilt of the three coordinate planes to the outside of their trihedral on the main chart plane. We also get the axonometric perspective from these main related views. With this we intend to simplify some graphic constructions of the axonometric system in order to improve the transmission of its scientific content.

2 Axonometric system fundamentals

To better explain and understand the communication, the fundamentals of orthogonal axonometric perspective are initially presented. We start from a trirectangular trihedral shown in fig. 1, called system trihedral or reference trihedral $(O)(x)(y)(z)$, formed by three lines or edges (x), (y) and (z) that intersect at a point called the vertex (O). On each of the lines and from the vertex we carry a magnitude unit, which establishes in each of the lines (x), (y) and (z) the points (I), (J) and (K) respectively. The planes $(I)(O)(J)$, $(J)(O)(K)$ and $(K)(O)(I)$ that form the sides of the system trihedral are called coordinate planes. A fourth plane Π, which does not contain any edge or side of system trihedral $(O)(x)(y)(z)$, passes by the vertex (O). This plane is called projection plane or main chart. If we project orthogonally (on the p_{Π} direction) the system trihedral $(O)(x)(y)(z)$ over the main chart plane Π we get straight away the axonometric perspective formed by the lines or axes x, y and z, intersecting at the vertex O coincident with the point (O). The parallelism and proportionality are preserved in the axonometric perspective. The points (I), (J) and (K) can be projected in a similar way, getting respectively points I, J and K. The magnitudes $\overline{OI}=u_x$, $\overline{OJ}=u_y$ and $\overline{OK}=u_z$ are called axonometric scales.

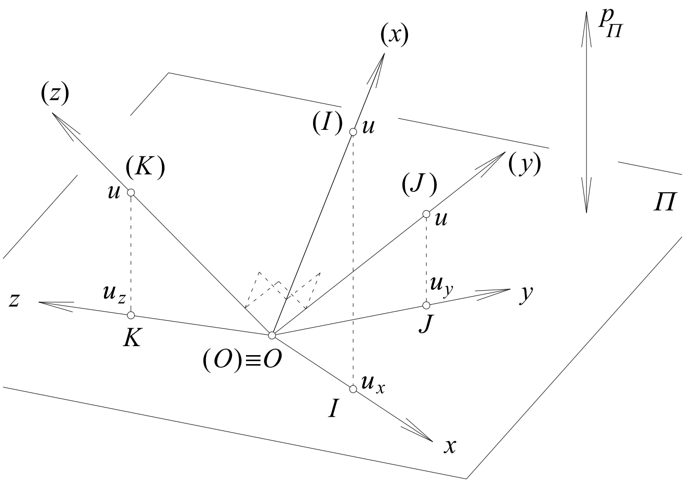

Fig. 1 Fundamentals of orthogonal axonometric system

In fig. 2 the angles forming the axis of the axonometric perspective $\alpha_z = \widehat{IOJ}$, $\alpha_x = \widehat{JOK}$ and $\alpha_y = \widehat{KOI}$ are represented. The different angles that can take projected axes of the trihedral system give rise to different kinds of axonometric perspectives: isometric (3 equal angles), diametric (only 2 equal angles) y trimetric (3 different angles).

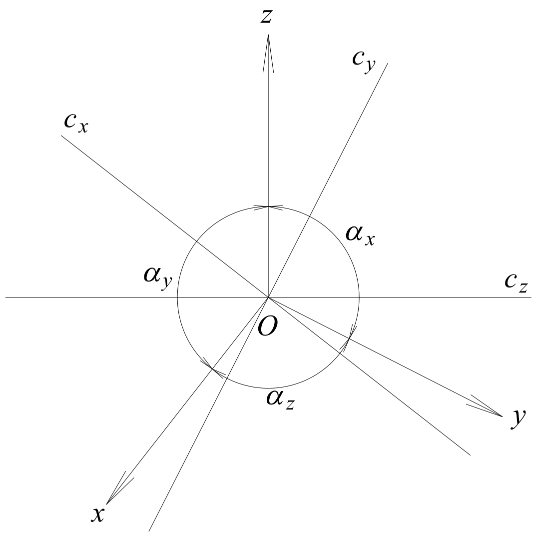

Fig. 2 Trihedral system intersection with the main chart plane

In addition, fig. 2 shows the intersection of the main chart plane Π with the coordinate plane $(I)(O)(J)$. This intersection is a line called $c_z = \Pi \cap (I)(O)(J)$ perpendicular to both the edge (z) and the projection direction p_Π, so it is also perpendicular to the axis z. Similarly we can determine the intersection lines between the main chart plane and the other two coordinate planes $c_x = \Pi \cap (J)(O)(K)$ and $c_y = \Pi \cap (K)(O)(I)$.

3 Trihedral system tilt on the main chart plane

The steps to get the trihedral system tilt on the plane of the chart Π is shown, in this communication, in the following two figures. Fig. 3 shows the tilt of a coordinate plane on the main chart plane. By any point I (orthogonal projection of point (I)) at a distance u_x from O over the axis x we draw a line parallel to the axis y. The intersection of this line with the line c_z determines the point P_z. The angle $\widehat{O(I)P_z}$ is straight like angle $\widehat{(I)O(J)}$, therefore three points O, (I) and P_z form a right triangle. This triangle is tilted down on the main chart plane. To do this, we draw a circle whose diameter passes through the points O and P_z. The intersection between this circle and the line through the point I and is perpendicular to the line c_z determines the tilted down point $(I)_z$. The distance $\overline{O(I)_z}$ measures a distance u and its direction determines the line $(x)_z$. Through the point O we draw a line $(y)_z$ perpendicular to $(x)_z$. These two lines represent the tilt of side $(x)O(y)$ of the trihedral of the system on the main chart plane Π.

If on the tilted-down line $(y)_z$ we bring the unit magnitude u from O we get the point $(J)_z$. The line through this point and perpendicular to the line c_z determines the point J on the axis y. The distance $\overline{OJ}$ determines the axonometric scale u_y on the axis y. Through a point J we draw a line parallel to the axis x, and we get on the line c_z the point Q_z. The angles forming the axonometric axis with the tilted-down ones are called $\beta_z = \widehat{IO(I)_z}$ and $\gamma_z = \widehat{JO(J)_z}$.

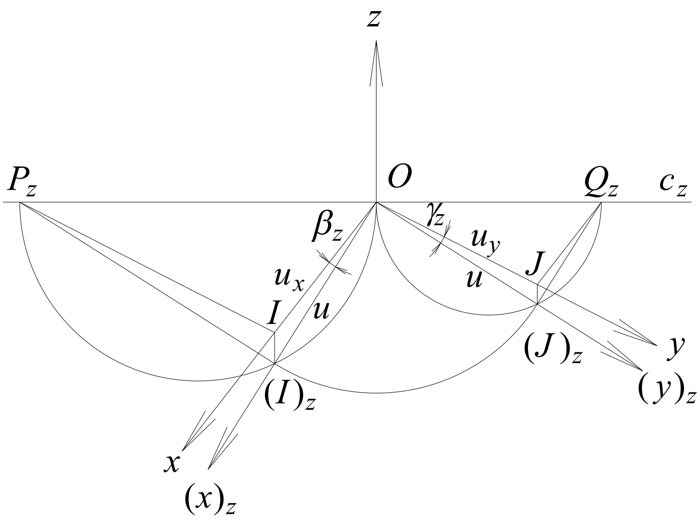

Fig. 3 Tilt of a coordinate plane on the main chart plane

In fig. 4 we continue developing the above procedure to show the tilts of the other two coordinated sides or planes that form the trihedral system on the main chart plane.

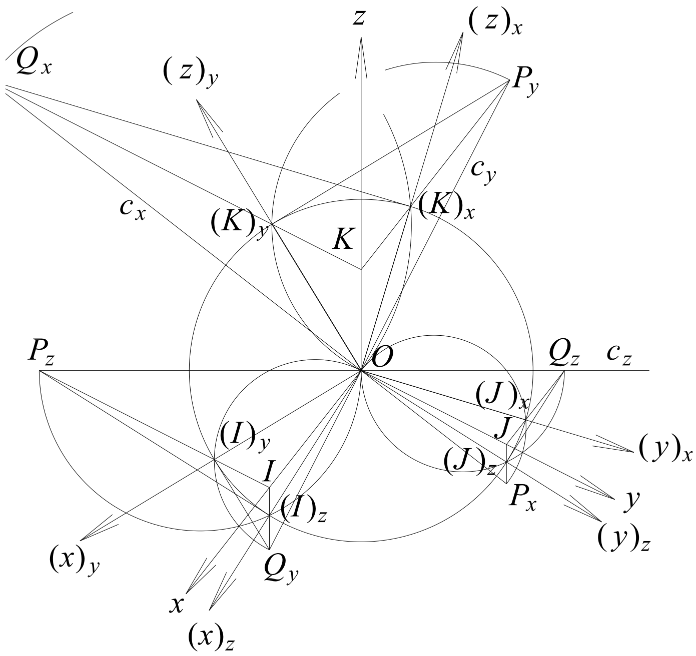

Fig. 4 Tilt of a trihedral system on the main chart plane

It can be seen in the previous figure that each edge of the trihedral system folds twice on the main chart plane. This is because each edge is associated to two coordinate planes.

4 Main related views

Fig. 5 shows the relationship between projected coordinate planes and tilted-down on the main chart plane. Related views are also represented in this communication are called main or trihedral views.

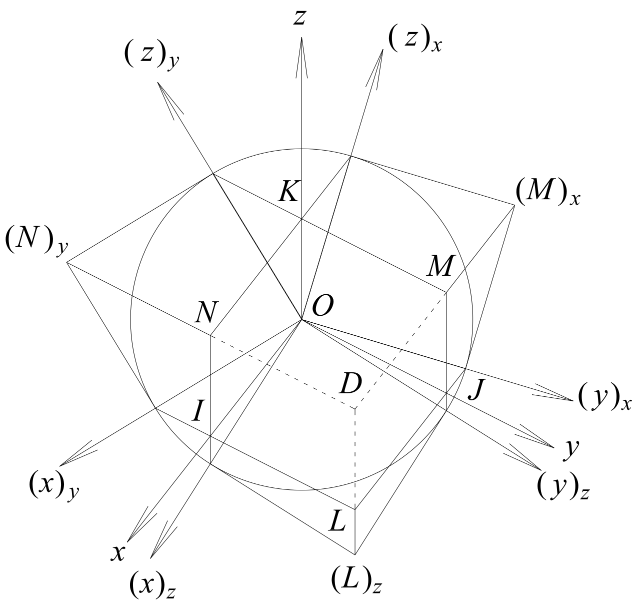

Fig. 5 Main related views

Trihedral views are obtained directly and are caused by the tilt of the sides of the trihedral system or coordinate planes on the main chart plane.

5 Axonometric system from the main related views

In order to derive from the main related views the axonometric axes with their scales, is first necessary to know how to determine the tilt on the main chart plane of a plane perpendicular to any of the three coordinate axes. The fig. 6 shows how to get the tilt of the plane perpendicular to the axis (z) and passing through point (D), on the main chart plane Π. It's observed that the lines LP_z and LQ_z are parallel to lines DR_z and DS_z respectively. In turn we can determine the line $d_z \equiv R_z S_z$, which is the intersection of the plane $R_z(D)S_z$ with the main chart plane and therefore parallel to the line c_z. The tilt on the main chart plane Π of the points (L) and (D) contained in planes perpendicular to axis (z) occurs at the same point $(L)_z$ or $(D)_z$.

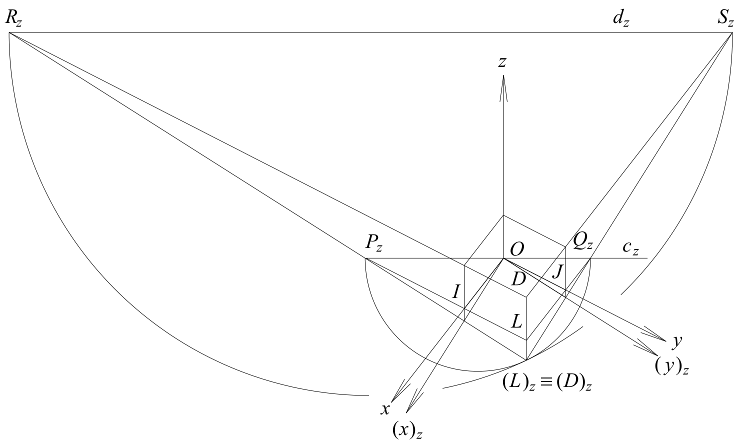

Fig. 6 Tilt of a plane perpendicular to the axis (z) on the main chart plane

In fig. 7 we perform the same operations with respect to the axis (x) than those presented in fig. 6. In this case we obtain the tilt on the main chart plane Π of points (M) and (D) contained in planes perpendicular to the axis (x). The determined point is the same $(M)_x$ or $(D)_x$. We have determined, also, the line $d_x \equiv R_x S_x$ which is the intersection of plane $R_x(D)S_x$ with the main chart plane and parallel to the line c_x.

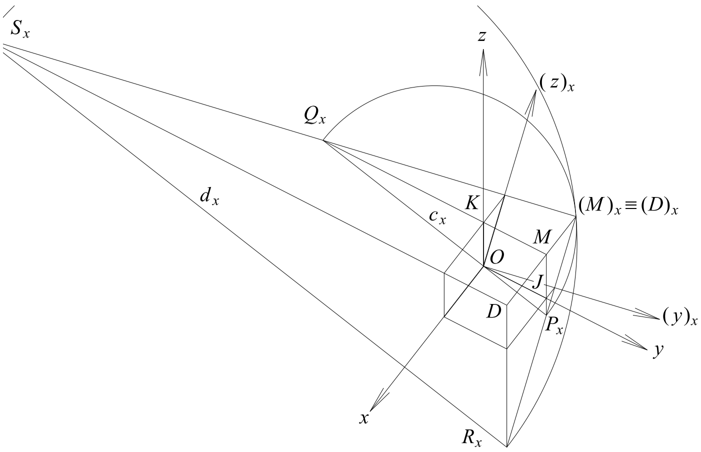

Fig. 7 Tilt of a plane perpendicular to the axis (x) on the main chart plane

In fig. 8 we perform the same operations with respect to the axis (y) than those represented in both fig. 6 and fig. 7 In this case we obtain the tilt on the main chart plane Π of points (N) and (D) contained in planes perpendicular to axis (y). The determined point is the same $(N)_y$ or $(D)_y$. We have determined, also, the line $d_y \equiv R_y S_y$ which is the intersection of plane $R_y(D)S_y$ with the main chart plane and parallel to line c_y.

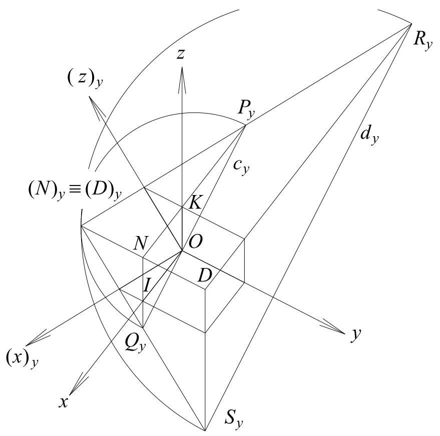

Fig. 8 Tilt of a plane perpendicular to the axis (y) on the main chart plane

Composing the three previous figures 6, 7 and 8 in the one fig. 9 one can deduce how to construct the orthogonal axonometric perspective from the main related views.

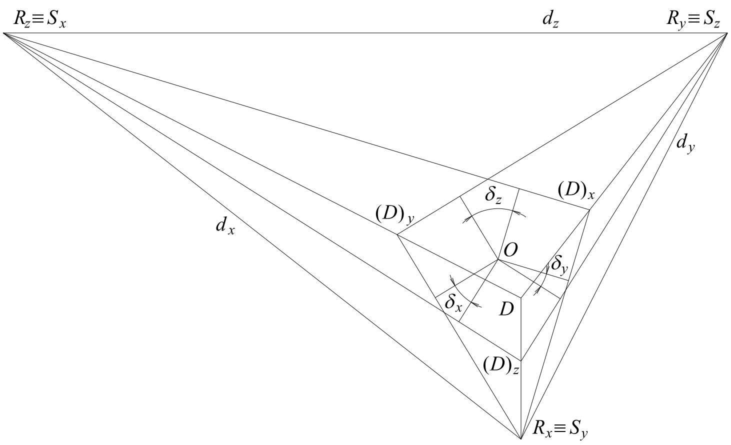

Fig. 9 Building of the axonometric perspective from the main related views

The three tilted-down plane on the main chart plane form the angles $\delta_x = \beta_z + \gamma_y$, $\delta_y = \beta_x + \gamma_z$ and $\delta_z = \beta_y + \gamma_x$ between them. The three axes of the projected system are x, y and z, pass through vertex O and are parallel to lines $DR_y \equiv DS_z$, $DR_z \equiv DS_x$ and $DR_x \equiv DS_y$ respectively.

6 Conclusions

Main related views or trihedral views have been defined as those produced by the tilt of the three coordinate planes to the outside of their trihedral on the main chart plane, which passes through the reference trihedral vertex. For each main projection plane there are some unique main related views, regardless of the direction of projection. From the main trihedral views you can get the axonometric perspective from which they came. We have presented some new graphic constructions to define the axonometric system, such as: determine the projections of the axes from the main related views and determine the axonometric scales from the axes. It is considered that this new definition of main related views, allows a simplification in the understanding of systems of representation and its use is suitable for teaching purposes.

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