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Parametric Modeling of Free-Form Surfaces for Progressive Addition Lens

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Abstract

Purpose:
By the mid-40s, the amplitude of accommodation has declined sufficiently so that most of us can no longer accommodate clearly and comfortably for close tasks. This condition, called presbyopia, is related to the growth of the lens.
Nowadays, due to the development of the free-form technologies, the progressive addition lenses (PAL) are the best solution for the presbyopia. To obtain the necessary optical properties, a continuous change in the curvature along a line (called corridor) must be provided on a surface of the lens (progressive surface).

Method:
In this work a method for the parametric design and analysis of progressive addition lens, based on discrete shape modeling, is proposed. Both the optical (e.g. power and addition) and the geometrical (e.g. inset, corridor length, amplitude of the distance and near vision area) parameters have been taken into account.

Result:
Different parameters are assessed in the design stage of PAL. The method developed for the analysis of surface optical properties, especially with regard to the astigmatic surface power, has proved an essential tool for the analysis of results. A key role in the resultant optical properties of the designed PAL is covered by the distribution of the curves in the intermediate area and by the curvature equation along the corridor.

Discussion & Conclusion:
The free-form technologies, recently introduced in the manufacturing process of ophthalmic lenses, allow the production of high performance and custom PALs. Surface power and astigmatic surface power show similar behaviour to other commercial progressive additional lens but, in addition, the designer can define the distribution of astigmatism in the intermediate region. Compared to the methods proposed in literature, this method shows more opportunities in the design parameters definition and allows highly customized lens, designed on the main habits of the wearer. Moreover the exchange data formats for the CNC manufacturing process were described.

1 Introduction

The ability of the eye to bring near objects into focus is called accommodation and it can be due to different mechanism as the change of corneal curvature, the change of lens power, the change of lens position along its axis or the change of the axial length of the eye, but the must probably mechanism is related to the curvature variation of the lens due to ciliary muscle action [1, 2].

By the mid-40s, the amplitude of accommodation has declined sufficiently so that most of us can no longer accommodate clearly and comfortably for close tasks [3]. This condition, called presbyopia, is closely related to the growth of the lens [4].

In order to correct this refractive disorder is possible to adopt different type of prosthetic devices as multifocal contact lenses [5], spectacle lenses for near vision, bifocal spectacle lenses, progressive spectacle lenses [6-8] and gradient index spectacle lens [9, 10]. Alternatively is possible to restore accommodation by means of surgery treatment e.g. scleral expansion and implantation of bifocal, multifocal or accommodating intraocular lenses (IOLs) [11-13].

Nowadays, due to the improvement of the free-form technologies, the progressive additional lenses are the best solution for the presbyopia for the majority wearers, with acceptance rates of ninety per cent or more [14]. This spectacle has a lens with a surface which is not rotationally symmetrical, with continuous change of focal power over a part or the whole of the lens [15]. In other words a progressive lens is a lens with a progressive surface (progressive side), with a continuous change of curvature over a part or whole of the surface [15]. In a progressive lens we can identify different zone [15, 16] (fig. 1):

- a distance portion having the dioptric power for the distance vision,
- a near portion or reading portion having the dioptric power for the near vision,
- an intermediate corridor that provides clear vision for ranges intermediate between distance and near,
- two blending regions that flanked to either side the other portions, which should have low the values of astigmatism and its gradient.

The surface power is proportional, through the refractive index, to the mean curvature of the surface, while the astigmatic surface power is proportional to the difference between the principal curvature [15]. In the distance and near vision portions and along the intermediate corridor the astigmatic surface power should be null and lower as possible in the blending regions. Besides, the variation of the astigmatic surface power in the blending regions should be smooth (the gradient should be lower as possible).

The points, called umbilics, in which the astigmatic surface power is null are the points in which the principal curvatures are equal. At these points all the normal curvatures are equal in all direction, and hence principal directions are indeterminate. Consequently the lines of curvature become singular at an umbilic [17].

Since the first progressive lens patent, filed in 1907 [18], to nowadays, several attempts to achieve progressive lenses and different geometrical modeling and manufacturing method have been proposed, but only in the sixties the first commercial success was achieved [14, 16].

In order to obtain a progressive surface is possible to adopt an elephant-trunk-shaped surface in which the point of a single vertical meridian that is umbilic point [19] (i.e. the principal curvatures are equal). For this progressive surface the rate of change in the unwanted astigmatism is twice the rate of change in addition power along the umbilic in accordance to the Minkwitz's theorem [20].

A variational method proposed by Loos et al. [21, 22] is based on the minimization of an error functional defined by an integral of two addenda: the first one is defined as the product of the difference of principal curvatures and a function that specifies how much an occurrence of astigmatism is penalized by the functional, while the second one is equal to the product of the difference between the mean curvature and the curvature necessary to achieve the desired power, and a function that weights the deviation from the required mean curvature. The method is implemented on B-spline patches. Other methods proposed by Loss [23, 24] are based on the surface modeling by fair reflection line pattern and on strategies derived from the evaluation of wavefront aberration for the optimization of the progressive surface.

Wang et al. [25-27]firstly analyzed the Loos' variational method and then proposed a numerical solution to the corresponding Euler-Lagrange equation (a fourth-order nonlinear elliptic partial differential equation). Other approach to the solution of the same problem was proposed by Otero et al. [28] that optimize the problem solution.

Regardless of design method, the performance assessment of a progressive lens or progressive surface can be measured by instruments based on mechanical contact as CMMs [29, 42] or on light transmission as focimeter and lens analyzer in which Hartmann-Shack [30, 31] or moiré deflectometry sensor [32-35] are adopted. The optical properties measured must be ranged in the ISO standard limit, that are defined for the progressive surface [36] and for the complete lens [37] as a function of the nominal power and astigmatic power in the reference point for distance and near vision. This limit is defined only to the design reference points for far and near vision, but no one prescription is defined for the corridor (fig. 1). In this regard an interesting method is proposed by Sheedy [32-35] and it is adopted to compare different progressive lens design. This studies show that actual progressive additional lens are connected to a specific design in which the amplitude of the distance and the near vision portion is constant.

Recent studies [38, 39] show that conditions as the task, the user lifestyle, the design of the progressive lens, the head and velocity movement are strongly related to the wearer adaptation and satisfaction. Subsequently, the need to design highly customized lens that take into account the wearer habits is necessary.

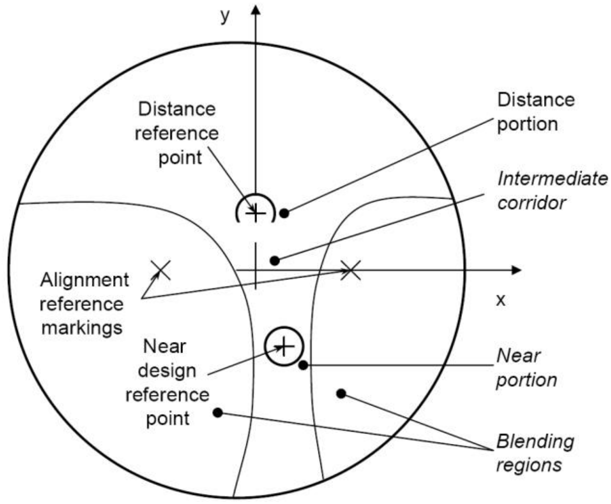


Fig. 1 Different zones in a progressive lens and relevant reference points.

The available commercial softwares allow only the opportunity to design PAL with the same astigmatic power distribution and with the same size and position of the distance and near vision portion. Consequently, in the commercial market of the PAL the name of a lens is related to its astigmatic surface power map [32-35].

In this work a parametric method for geometric modeling of progressive addition lens is proposed, which enables to control the surface power, the addition power, the corridor length and the amplitude of the aspheric surface relevant to near and distance portions. The method, based on the discretization of curves with known curvature, allows to design progressive lenses customized on the wearer habits and characteristics (occupation, age, head movements and relevant velocity, etc.). This result can be accomplished by changing the dimension and the position of the distance and near vision portion.

One of the main advantages of the proposed method is the ability to obtain different astigmatic surface power distribution in the same solution. Moreover the ability to change the dimension of the distance and near vision portion and of other parameters as the corridor length, enables a high customization based on the main habits of the wearer [38, 39]. Consequently, the proposed design method allows a high adaptation and satisfaction of the wearer.

In addition, compared to the methods in the literature, the proposed approach gives similar results without employing complex functions such as differential or integral equation [21, 28].

The proposed method can be simply integrated in the CAD-CAM software developed [40, 41] and the geometric model of the progressive surface can be saved in the typical exchange format provided by the new freeform technologies [14, 41].

Moreover surface optical properties of the designed progressive lens are shown according to a method described in previous works [29, 42].

All the algorithms, implemented in a plug-in for Rhinoceros V4.0 and V5 Wip, are available at the web site http://sites.google.com/site/lentiprogressive/Progressive_Lens_Design.

2 Method

The proposed modeling process involves the follow steps which are implemented in a plug-in:

- geometric and optic parameters setting,
- lens shape modeling
- geometric models export and analysis.

The modeling method is based on a discrete definition of the points on the surface and is outlined in figure 2.

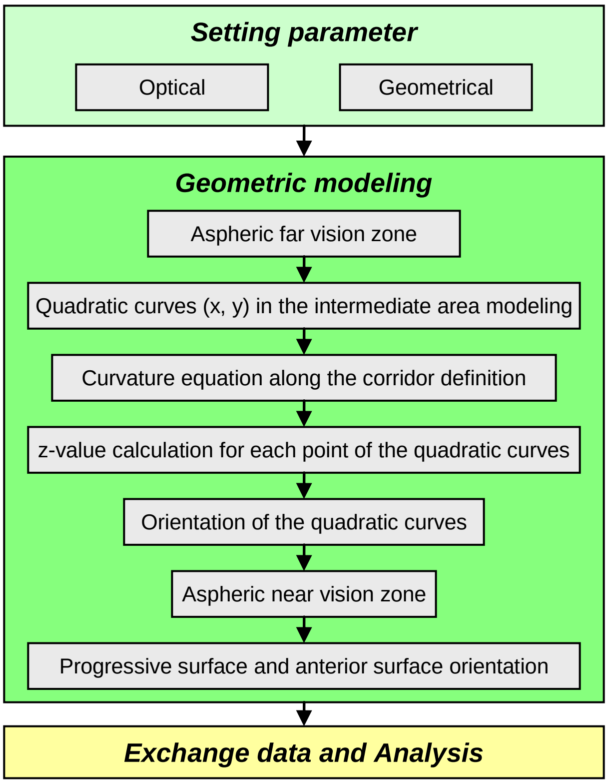


Fig. 2 Flow chart of the modeling process.

2.1 Setting parameters

The parameters involved into the shape modeling process of a progressive lens can be categorized into geometrics, which define the position and the dimension of distance and near vision zone, and into optical parameters from which the curvature radius of the optical zone is derived.

2.1.1 Geometric parameters

Fig. 3 together with tab. 1 explain the geometric parameters involved, where O is the lens centre.

The shape of the far and the near zone are delimited by a quadric equation in the x-y plane respectively trough the point DP1, DRP, DP2 and NP1, NRP, RP2:

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} a_2 \cdot t^2 + a_1 \cdot t + a_0 \\ b_2 \cdot t^2 + b_1 \cdot t + b_0 \end{pmatrix} \qquad (1)$$

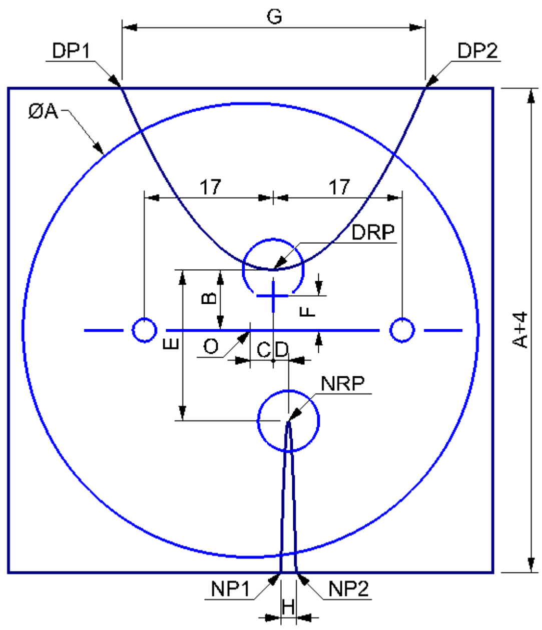


Fig. 3 Parameters for setting geometric properties (dimension in mm).

Dim.	Correspond to
A	Lens diameter
B	Y-coordinate of distance reference point
C	Off-center (X-coordinate of distance reference point)
D	Inset (X-distance between distance and near reference point)
E	Corridor height
F	Fitting cross
G	Amplitude of the edge distance zone
H	Amplitude of the edge near zone

Tab. 1 Meaning of dimension represented in fig. 3.

2.1.2 Optical parameters

The optical parameters required for the modeling of the lens are summarized in tab. 2.

By the P_{FS} [D], relevant to an ideal lens with refractive index n=1.523 (i.e. the n of a crown glass, traditionally employed in the manufacturing process of lenses), the curvature radius of the front surface R_{FS} [mm] can be derived:

$$R_{FS} = 523 / P_{FS} \qquad (2).$$

Assuming a first attempt value for centre thickness CT, it is possible to derive the radius of curvature for the distance zone R_D [mm] and for the near zone R_N [mm]:

$$R_D = 1/c_D = 1 \Big/ \left(\frac{n}{n \cdot R_{FS} - CT \cdot (n-1)} - \frac{P}{1000 \cdot (n-1)} \right) \qquad (3),$$

$$R_N = 1/c_N = 1 \Big/ \left(\frac{n}{n \cdot R_{FS} - CT \cdot (n-1)} - \frac{P + Add}{1000 \cdot (n-1)} \right) \qquad (4).$$

Moreover the designer can change the radii of curvature for the front surface, for the distance and for the near portions. Moreover it is possible to introduce aspheric coefficients for the distance zone K_D and for the near zone K_N, in order to reduce the out-axis astigmatism.

Acronym	Parameter
P	Power [D]
Add	Addition [D]
n	Refractive Index
P_{FS}	Front surface power (n=1.523) [D]
K_D	Distance aspheric coefficient
K_N	Near aspheric coefficient

Tab. 2 Optical parameters.

2.2 Lens shape modeling

The proposed method performs the design of the back surface (the surface closest the eye) of a progressive addition lens.

The distance and the near vision area are delimited by a quadratic curve. To ensure a continuous variation of the optical power along the corridor with low astigmatic surface power, the principal curvatures must be equal (i.e. the points of the corridor must be umbilics [17]).

A specific modeling procedure was developed to provide this condition:

1) The distance zone was modelled as a standard aspheric surface centred in the DRP.
2) A set of quadratic curves (as eq. 1) which covers the area between the distance and the near portions (intermediate area) was drawn in the x-y plane.
3) A curvature equation along the corridor is defined.
4) For each point ($x_{j,i}$,$y_{j,i}$) of the i^{th} curve (sampled with a constant step) the $z_{i,j}$ value is derived by the equation (aspheric surface centered in xc, yc):

$$z_{i,j} = \frac{\left((x_{i,j}-xc_i)^2+(y_{i,j}-yc_i)^2\right)}{Rc_i+\sqrt{Rc_i^2-Kc_i\cdot\left((x_{i,j}-xc_i)^2+(y_{i,j}-yc_i)^2\right)}} \quad (5)$$

   where xc_i, yc_i are the Cartesian coordinates, Rc_i is the radius of curvature at the centre and Kc_i is the aspheric coefficient at the point (xc,yc) of the corridor.
5) For each point ($x_{i,j}$,$y_{i,j}$,$z_{i,j}$) of the curve i^{th} a rotation and a translation were made in order to align the normal of the eq. 5 in the point ($x_{i,j}$=xc_i, $y_{i,j}$= yc_i) with the normal of the $(i-1)^{th}$ curve in the same point.
6) The near zone, delimited by the quadratic curve, was modelled as a standard aspheric surface centred in the NRP and than aligned with the last calculated normal.
7) Point 1 to 6 described the shape modeling of the back surface of the lens which must be aligned to the front surface taking into account the “prism-thinning” to minimize lens thickness and weight.

The distance zone is sampled with constant step as shown in fig. 4 and the z-coordinates are derived from the eq. 5 where xc_j = C, yc_j = B, Kc_i = K_D and Rc_i = R_D.

Two different approaches were evaluated for the construction of curves in the intermediate area.

In the first approach the quadratic curves were obtained trough the interpolation of 3 points as shown in fig. 5. For the i^{th} curves the first point is obtained by an uniform sampling of the corridor:

$$\begin{pmatrix} xc_i \\ yc_i \end{pmatrix} = \begin{pmatrix} C + D\cdot i/n \\ B - E\cdot i/n \end{pmatrix} \qquad i = 0,1,2,\ldots,n1 \qquad (6)$$

with n1 equal to the number of curves in the intermediate zone. The other 2 points, symmetrical to the y-axis, are derived with a displacement from the first point along a line angled of $\alpha i=i\cdot\delta=i\cdot\alpha/n1$ (with reference to the first line) at a distance equal to Li=LD+(LN-LD)·i/n. In fig. 5 at the left is explained the points identification and at the right the relevant curves.

Instead, in the second approach, the quadratic curves in the intermediate area are obtained by the interpolation of different points: the first point is the same, while hte second and the third point have been obtained sampling with regular step half circumference which lies between the extremes of the limit curves of the distance and near vision portions, with centre in the mid point. Fig. 6 shows the comparison between the two approaches: the second one has a higher density of the curves in the central intermediate zone than the first.

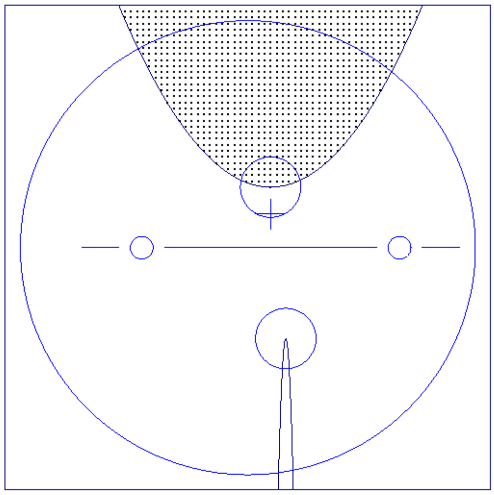


Fig. 4 Sampling of the distance zone.

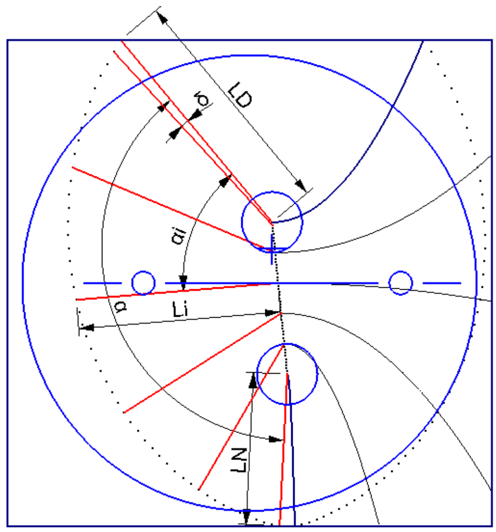


Fig. 5 First approach for the construction of the curve in the intermediate zone.

Different equations were assessed to define the radius of curvature Rc_i along the corridor. To obtain a continuous variation of optical properties the following equations of the curvature c_i=1/Rc_i are taken into account (curvature is proportional to the surface power and is related to the astigmatic surface power [15, 20, 29]):

$$c_a(t) = -2\cdot(c_N - c_D)\cdot t^3 + 3\cdot(c_N - c_D)\cdot t^2 + c_D \quad (7a),$$

$$c_b(t) = -3\cdot(c_N - c_D)\cdot t^4 + 4\cdot(c_N - c_D)\cdot t^3 + c_D \quad (7b),$$

$$c_c(t) = 6\cdot(c_N - c_D)\cdot t^5 - 15\cdot(c_N - c_D)\cdot t^4 + 10\cdot(c_N - c_D)\cdot t^3 + c_D \quad (7c),$$

$$c_d(t) = 10\cdot(c_N - c_D)\cdot t^6 - 24\cdot(c_N - c_D)\cdot t^5 + 15\cdot(c_N - c_D)\cdot t^4 + c_D \quad (7d),$$

$$c_e(t) = -20\cdot(c_N - c_D)\cdot t^7 + 70\cdot(c_N - c_D)\cdot t^6 + \\ -84\cdot(c_N - c_D)\cdot t^5 + 35\cdot(c_N - c_D)\cdot t^4 + c_D \quad (7e),$$

$$c_f(t) = -35\cdot(c_N - c_D)\cdot t^8 + 120\cdot(c_N - c_D)\cdot t^7 + \\ -140\cdot(c_N - c_D)\cdot t^6 + 56\cdot(c_N - c_D)\cdot t^5 + c_D \quad (7f)$$

where t=i/n $\in$ [0, 1]. These polynomial functions have been derived adopting the constraints on the derivative summarized in tab. 3.

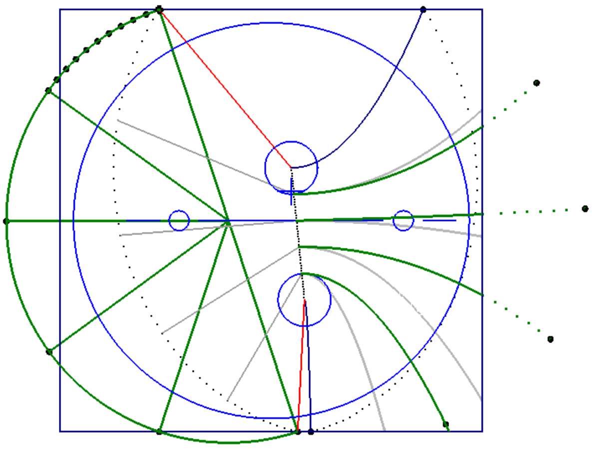


Fig. 6 Second approach (green) for the construction of the curves in the intermediate zone and comparison with the first one (grey).

Eq.	Constraints
7a	$c(0) = c_D,\ \frac{dc(t)}{t}\Big\vert_{t=0} = 0,\ c(1) = c_N,\ \frac{dc(t)}{t}\Big\vert_{t=1} = 0$
7b	As 7a and $\frac{d^2c(t)}{t^2}\Big\vert_{t=0} = 0$
7c	As 7b and $\frac{d^2c(t)}{t^2}\Big\vert_{t=1} = 0$
7d	As 7c and $\frac{d^3c(t)}{t^3}\Big\vert_{t=0} = 0$
7e	As 7d and $\frac{d^3c(t)}{t^3}\Big\vert_{t=1} = 0$
7f	As 7e and $\frac{d^4c(t)}{t^4}\Big\vert_{t=0} = 0$

Tab. 3 Constraints assumed to define the radius of curvature along the corridor.

The equation assumed for the aspheric coefficient along the corridor was derived using constrains similar to eq. 7a obtaining:

$$Kc_i = -2\cdot(K_N - K_D)\cdot t^3 + 3\cdot(K_N - K_D)\cdot t^2 + K_D \quad (8).$$

Fig. 7 shows the results obtained following the procedure described in the numbered list from point 1 to 5 for modeling the progressive surface.

A main final step in the lens design is the mutual position and orientation between the front (spherical) and the back (progressive) surface: position must ensure the minimum thickness at the edge (about 1 mm) for the positive lens and at the centre for the negative (about 2 mm); orientation must assure the direction of the visual axes [43, 44]. These characteristics are strongly related to the off-center (which must be defined in the geometric parameters setting step) and to the prism-thinning which has been used for many years to improve the cosmetic appearance of progressive lenses by generally applying base down prism [45].

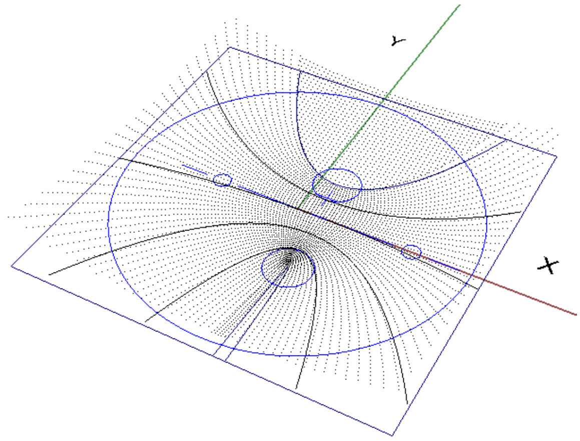


Fig. 7 Example of the result of the progressive surface modeling procedure.

When no prism is prescribed, a simple way to obtain the prism-thinning is to rotate the progressive surface along the x-axis in order to get the same z-coordinate at the points (C, A/2) and (C+D, -A/2). Then, a spherical surface (front surface) with radius R_{FS} and centre (C,0,R_{FS}) can be drawn. Subsequently the condition on the thickness mentioned above must be obtained by the necessary translation. Finally, to meet the needs of the manufacturing process, the normal at the origin of the front surface must be aligned to the z-axis and the z-coordinate of the progressive surface at the origin should be set equal to zero. An example of a section of a progressive lens model is shown in fig. 8. In the figure a mesh is represented, derived by the cloud of points with the Delaunay algorithms [46], using a free plug-in available on line at [47].

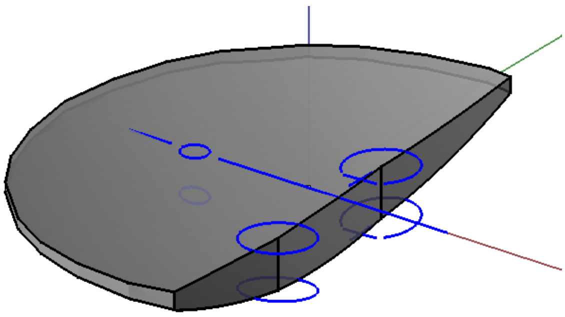


Fig. 8 Geometric model of a progressive lens obtained with the proposed procedure, sectioned along the corridor.

2.3 Data exchange format and geometric model analysis

Data exchange format required a constant step evaluation of the z-coordinates along the x- and y-axis. The z-values are obtained by fitting the calculated points with a local general second-order polynomial with six coefficients [29, 40, 48]. The local fitting involved all the points having the x-y coordinate inside a circular neighbourhood with a dimension defined in the export and analysis stage. The same coefficients allow to estimate the curvature properties and subsequently the surface optical properties of the progressive surface.

2.3.1 Data exchange format

The free-form manufacturing process of PAL uses CNC machines tools for turning, milling, grinding and polishing. Manufacturers of such machine tools (e.g. OptoTech, Schneider, Satisloh) require that the geometric models of the lenses are provided with special formats.

There are two main formats for data exchange and both require that the surface of the lens is defined by a cloud of point with a constant step along the x-axis and y-axis in a square area.

The first format has a header in which are summarized the step along the x and y-axis in mm (Interval=…) and the number of step along each axis (Count=…). Under the header for every line there is the z-coordinate, comma separated, of each point along y-axis with constant x-coordinate. The extension of this format is *.hmf.

The second is similar to the ascii points format in which every line takes the form of the x-y-z Cartesian coordinates of a point separated by tabulation or space.

An example of the formats for data exchange between CAD and CNC are shown in tab. 4.

Data exchange format	Extension
File Version=1.2 [Properties] Count=77 Interval=0.8 [Data] 3.723,3.578,3.436,3.299,… 3.674,3.529,3.388,3.250,… …	*.hmf
-30.4 30.4 3.723 -30.4 29.6 3.578 -30.4 28.8 3.436 -30.4 28.0 3.299 -30.4 27.2 3.165 -30.4 26.4 3.036 …	*.xyz

Tab. 4 Example of data exchange formats.

2.3.2 Surface optical properties analysis

In the design of a progressive addition lens is essential to derive the surface optical properties [22, 29, 42]. Surface astigmatism SA and surface power SP especially play a key role in the acceptance rates of a PAL.

These surface optical properties SA and SP can be derived from principal curvatures k_1 and k_2 by the refractive index n [29, 42] as:

$$SA = (n-1)\cdot(k_1 - k_2) \qquad (9a),$$

$$SP = (n-1)\cdot\frac{k_1 + k_2}{2} \qquad (9b),$$

With the same coefficients calculated from the second-order polynomial fitting for the estimation of the z-coordinates, it is possible to estimate the principal curvature in the same points of the data exchange, as described in [29, 41, 42].

Consequently, after the Delaunay triangularization of the points describing the optical properties [47], SA and SP can be represented as 3D surfaces. Moreover the contour lines, based on cross sections at every 0.50 D, representing the curves having the same surface optical properties, have been proposed in order to understand the results and compare it with literature.

3 Results and discussion

The method described is implemented in a plug-in for Rhinoceros 4.0 or 5 Wip in which optical and geometrical input parameters can be defined by specific list box as in fig. 9 a and b. Trough eq. 2-4 radii of curvature of the front surface, of the distance vision area and of the near vision portion can be derived and proposed to the designer who can change radii and aspheric coefficients in the relevant list box (fig. 9 c). Therefore the parameters for the data exchange can be set as in fig. 9 d.

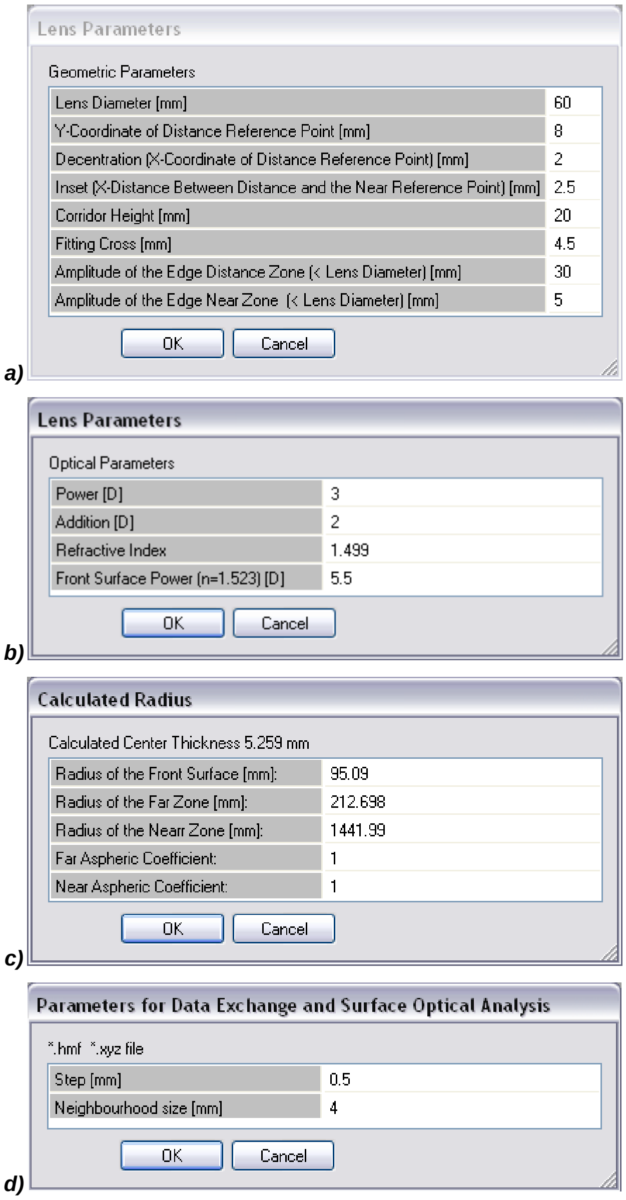


Fig. 9 List boxes for the input parameters: a) geometric, b) optical, c) derived radius of curvature and aspheric coefficients, d) data exchange.

Following the procedure described, the plug-in computes shape and optical properties which can be visualized as 3D mesh. Moreover the relevant files in *.hmf and *.xyz format are performed and exported in a folder named C:\Progressive for the CNC machining. In fig. 10 an example of the results is shown where iso-astigmatic and iso-power contour lines every 0.50 D are emphasized in the top view, while the shape of the lens (front surface and back surface), astigmatic surface power (cylinder map) and surface power (power map) are highlighted as mesh in the perspective view. Astigmatism and power map is scaled in the z-axis 5 times. Moreover the representation is subdivided in different layers which enclose respectively the geometric design, the lens shape, the cylinder map (astigmatic surface power) and the surface power map.

To assess the performance of the method eight geometric models, derived by the parameters in tab. 5, are analyzed with the virtual tools described in 2.3: fig. 11 illustrates the curvature and the relevant derivative trend along the corridor, fig. 12 shows the astigmatic surface power and the surface power, while tab. 6 reports the maximum value of the astigmatic surface power at the left and at the right of the corridor near the x axis and the surface power in the distance vision portion and in the near vision area. In these test cases the influence of the approach for drawing the quadratic curve in the intermediate area, the curvature equation along the corridor, the surface power and the addition are studied.

It is possible to verify that adopting the second approach (which has a higher density of the curves in the central zone of the intermediate portion than the first as described in 2.2) a smoother increase of astigmatism is obtained. In general this is a positive result, but the habits of the wearer and the shape of the frame should be considered too. Furthermore, the maximum value of the surface astigmatic power, which is higher in the second approach must be taken into account in the design stage.

The different curvature equation along the corridor allow to control the astigmatism map variation: when higher order polynomials were adopted (which guarantee null high order derivative at the starting and ending points of the curvature along the corridor) a smoother variation of the astigmatic surface power together with higher values of maximum astigmatism have been obtained.

Decreasing the minimum value of the derivative of the curvature, adopting the same distribution of the curves in the intermediate zone, it was observed that the maximum value of the astigmatic surface power increases.

Moreover, the derivative of the curvature along the corridor is an important parameter: it is proportional to the derivative of the power along the corridor, which is related to the derivative of the surface astigmatic power along the perpendicular direction (Minkwitz theorem [20]).

Comparing fig. 12 b) together to fig. 12 g), in which the only the addition of the lens is changed, a proportional variation in the astigmatism map has been found. Same considerations can be drawn from tab. 6: the ratio between the maximum surface astigmatism is equal to the ratio between the addition (test cases b), g)).

	Parameter	Test cases							
		a)	b)	c)	d)	e)	f)	g)	h)
Geometric	A	60 mm							
	B	8 mm							
	C	2 mm							
	D	2.5 mm							
	E	20 mm							
	F	4.5 mm							
	G	20 mm							
	H	5 mm							
Optic	P	3.00 D							-3 D
	Add	2.00 D						3 D	2 D
	n	1.499							
	P_{FS}	5.5 D						6.5 D	1 D
	K_D, K_N	1							
Corr. eq.		7a	7b	7c	7d	7e	7f	7b	7b
Int. curve		2^{st}	2^{st}	1^{nd}	1^{nd}	1^{nd}	1^{nd}	2^{st}	2^{st}

Tab. 5 Parameters adopted in the test cases.

Comparing fig. 12 b) together to fig. 12 h), in which the only the power of the lens is changed, it was observed that no significant change in astigmatism map has been found. Moreover in tab. 6 the same value of the maximum surface astigmatism can be found for the test cases b), h).

By other test cases it is possible to verify that:

- increasing the amplitude of the distance vision portion, the maximum astigmatic surface power increases,

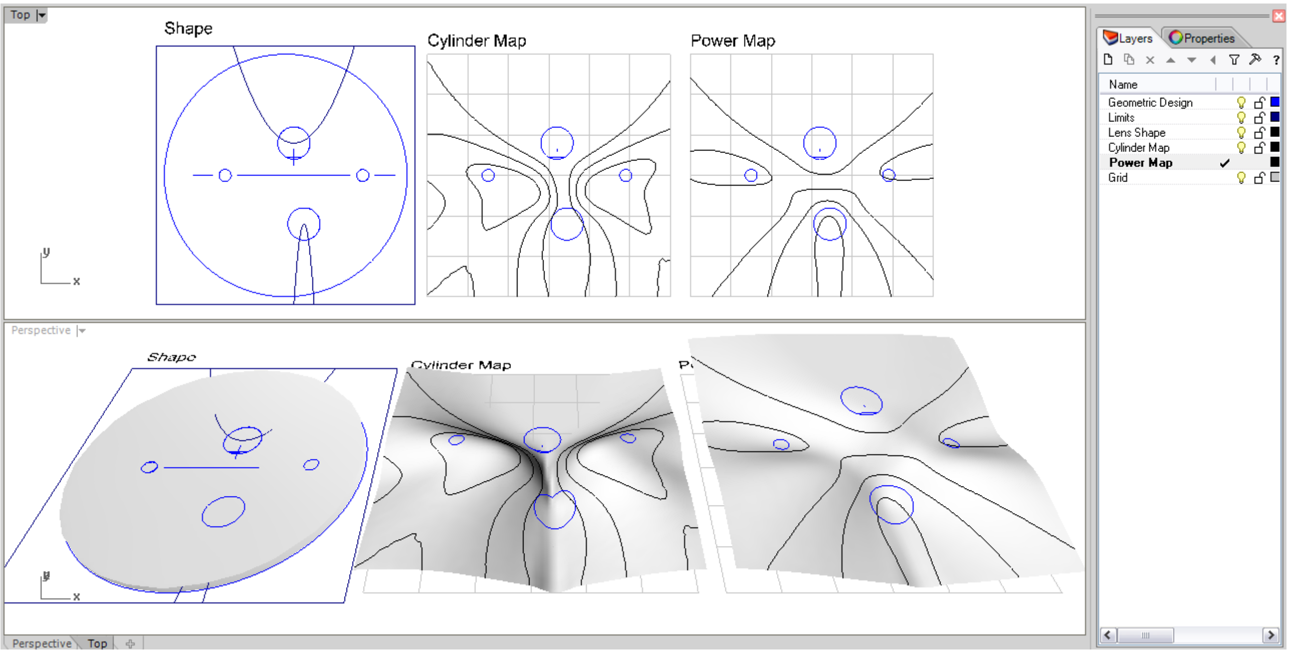
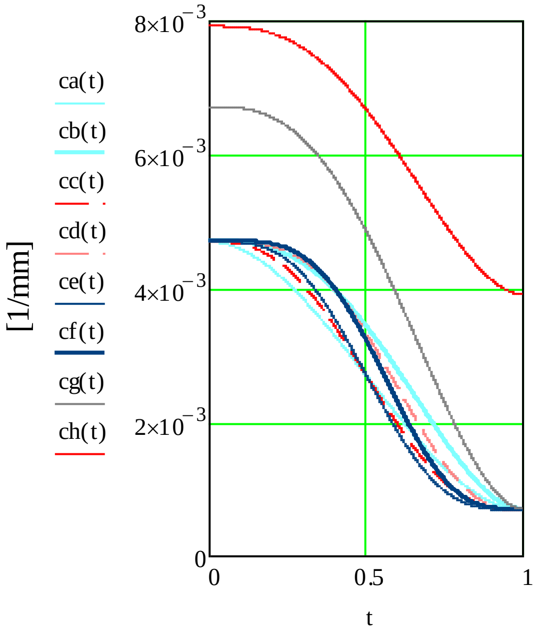


Fig. 10 Example of shape, astigmatism map and power map of a progressive lens performed with the proposed method.

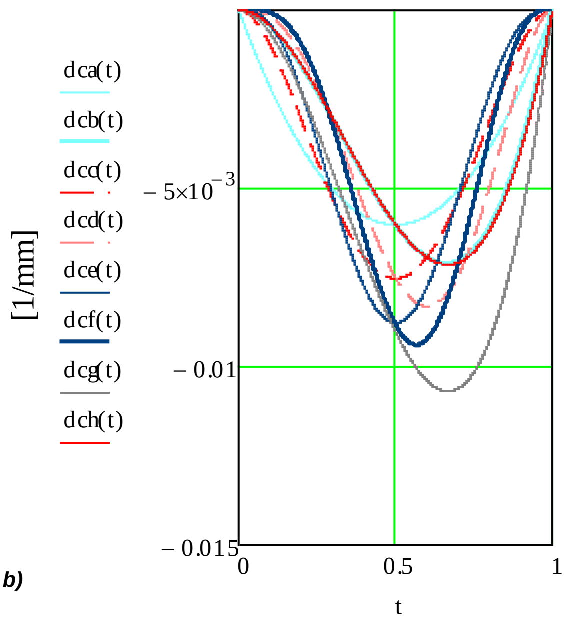
- increasing the amplitude of the near vision area, the maximum astigmatic surface power increases,
- decreasing the corridor length, the maximum astigmatic surface power increases.

The off-center parameter is necessary to reduce the work-piece dimension, because the portion of the lens near to the nose is smaller than the portion near to the temple, while the inset parameter is related to the power and to the addition of the lens [6].

Refractive index is related to the material for the lens manufacturing. Increasing the refractive index the thickness can be reduced and the curvature radius of the front surface can be increased. Consequently the volume of the lens (and the relevant weight) decreases and the cosmetic appearance of the lens improves. Changing the refractive index no significant change in surface astigmatic power appears.



a)



b)

Fig. 11 Curvature a) and relevant derivative b) along the corridor corresponding to the parameters in tab. 5.

	a)	b)	c)	d)	e)	f)	g)	h)
Max Surf. Ast. R [D]	1.84	1.92	1.58	1.64	1.84	1.89	2.87	1.92
Max Surf. Ast. L [D]	1.82	1.89	1.57	1.57	1.82	1.82	2.83	1.89
Surf. Pow. Dist. [D]	-2.35	-2.35	-2.35	-2.35	-2.35	-2.35	-3.36	-3.96
Surf. Pow. Near [D]	-0.35	-0.35	-0.35	-0.35	-0.35	-0.35	-0.36	-1.96

Tab. 6 Maximum value of the astigmatic surface power at the left and at the right of the corridor near the x axis and the surface power in the distance vision portion and in the near vision area relevant to the parameters in tab. 5.

Aspheric coefficients allow to minimize the side effects that are associated with their wear as the oblique astigmatic error or the mean astigmatic error [7, 8].

Compared to the methods in the literature, the proposed approach gives similar results without employing complex functions such as differential or integral equation [21, 28]. In the commercial market of the progressive spectacles the name of a lens is related to its astigmatic surface power map [32-35]. One of the main advantages of the proposed method is the ability to obtain different astigmatic surface power distribution in the same solution. Moreover the ability to change the dimension of the distance and near vision portion and of other parameters as the corridor length, enables a high customization based on the main habits of the wearer [38, 39]. Consequently, the proposed design method allows a high adaptation and satisfaction of the wearer.

4 Conclusion

In this work a method for the parametric design and analysis of progressive addition lens, based on discrete shape modeling, has been proposed. Both the optical (e.g. power and addition) and the geometrical (e.g. inset, corridor length, amplitude of the distance and near vision area) parameters have been taken into account. Moreover the exchange data formats for the CNC manufacturing process were described.

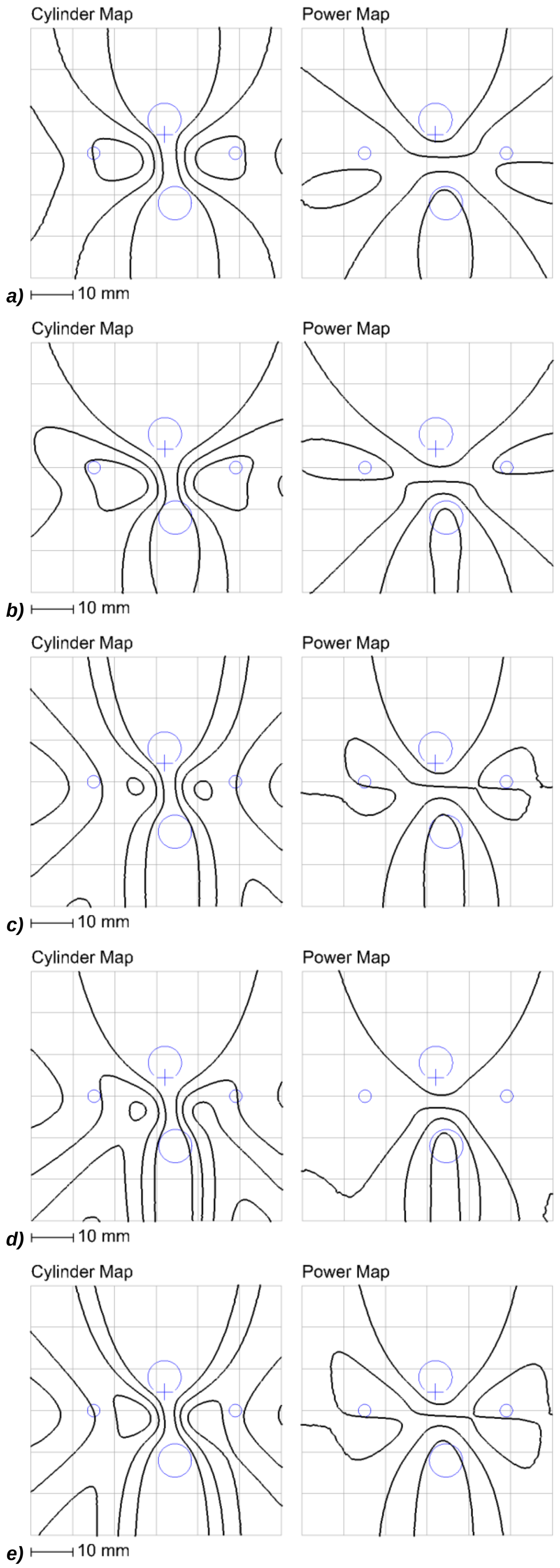
The distance and the near vision portions are defined as aspheric surfaces delimited by quadratic curves. The intermediate zone are defined as a series of quadratic curves in the x-y plane which cover homogeneously this area and assume a z-coordinate value as an aspheric with a continuous variation of curvature.

Two different approaches were evaluated to cover the intermediate zone and different equations for the curvature along the corridor were analyzed. Instead of quadratic curve any other type of curve can be adopted to delimit the optical zone and cover the intermediate area. Similarly other equations for the curvature along the corridor can be adopted.

The described method is currently applied to the design process of glass mould for PAL.

With minor improvements the proposed method is able to define the shape of progressive lens with prism or astigmatism prescription.

Surface power and astigmatic surface power show similar behaviour to other commercial progressive additional lens. Compared to the methods proposed in literature and available in the market, this approach shows more opportunities in the design parameters definition, such as corridor length, amplitude of the far vision and near vision area, and consequently allows highly customized lens, designed on the main habits and characteristics of the wearer.



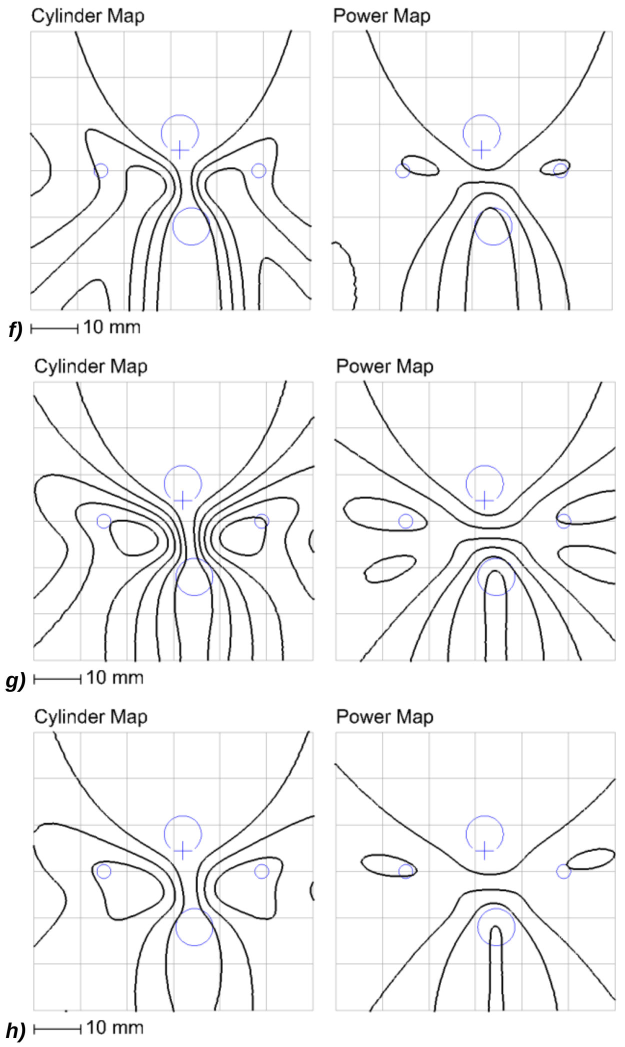


Fig. 12 Surface astigmatic power and surface power map relevant to the parameters in tab. 5. The figures show Iso-astigmatism and iso-power curves with step of 0.50 D.

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